Assignment 2. Due Wednesday, Jan. 26.

Reading: Horn and Johnson, secs. 1.0–1.3, 1.5.
Trefethen and Embree, secs. 1–2.

1. **(a)** Determine precisely the field of values of the n by n Jordan block with eigenvalue λ:

$$J = \begin{pmatrix} \lambda & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda \end{pmatrix}.$$

[Hint: Use the result from problem 2 of HW 1 and the fact that the eigenvalues of an n by n real symmetric tridiagonal Toeplitz matrix with a on the main diagonal and b on the sub- and super-diagonal are $\mu_j = a + 2b\cos(\pi j/(n+1))$, $j = 1, \ldots, n$. For a derivation of this formula see, for example, Iserles, *A First Course in the Numerical Analysis of Differential Equations*, Lemma 10.5, p. 198.]

(b) Let J be a 40 by 40 Jordan block with eigenvalue -0.6. What can you say about the behavior of $\|e^{tJ}\|_2$ as $t \to \infty$? Initially (i.e., near $t = 0$), does $\|e^{tJ}\|_2$ increase or decrease with increasing t? What is $d/dt(\|e^{tJ}\|_2)|_{t=0}$?

2. Show that for any first degree polynomial $p(z) = c_0 + c_1 z$ and any n by n matrix A,

$$\max_{z \in \mathcal{F}(A)} |p(z)| \leq \|p(A)\|_2 \leq 2 \max_{z \in \mathcal{F}(A)} |p(z)|.$$

[Crouzeix's conjecture is that for any polynomial p, $\|p(A)\|_2 \leq 2\max_{z \in \mathcal{F}(A)} |p(z)|$.]

3. Write a code to compute and plot the level curves of the resolvent norm $\|(zI - A)^{-1}\|$ over a user-specified region of the complex plane. Allow the user to specify the norm as well, say, the $1-$, $2-$, or $\infty-$norm. Use your code to compute pseudospectra in different norms for the matrix from problem 3 of HW 1 (i.e., a 50 by 50 matrix with -1's on the first subdiagonal, 1's on the main diagonal and the first three superdiagonals, and zeros elsewhere). Turn in a plot of your results. [You may use the MATLAB command `contour` to make your contour plots.]

4. Go to the *Pseudospectra Gateway* web site

`http://web.comlab.ox.ac.uk/projects/pseudospectra/`

and browse to see what is there. Then click on `Software` and `Eigtool` and download the Eigtool software and see if you can install it on your computer. If you are successful, you should be able to check your code in problem 3 by using Eigtool to compute 2-norm pseudospectra of the matrix for that problem. Let me know if you have problems getting Eigtool to work.