Assignment 5. Due Friday, May 8.

Reading: Through Difference Methods for the Cauchy Problem in the Notes.

1. Complete any remaining problems on Problem Set 2 in the Notes.
2. (1996 prelim, problem 7) Find all C^2 functions $u(x, y)$ which satisfy

$$(\partial_x - 2i)^2 u + (\partial_y - 2i)^2 u = 0$$

   and such that there exists $M > 0$ with

$$|u(x, y)| \leq M(1 + |x|^2 + |y|^2)$$

   for all $(x, y) \in \mathbf{R}^2$. Hint: Consider $\hat{u}$.

3. (2001 prelim, problem 7) Let L denote the following differential operator in $\mathbf{R}^n$:

$$L = \sum_{i=1}^{n} \alpha_i \frac{\partial^2}{\partial x_i^2},$$

   where each constant α_i is nonnegative. Denote by $\mathcal{S}(\mathbf{R}^n)$ the Schwartz space. Let $f \in \mathcal{S}(\mathbf{R}^n)$. Prove that there exists a unique $u \in \mathcal{S}(\mathbf{R}^n)$ such that

$$Lu - u = f.$$