Math 498 Chartier/Greenbaum
Homework #5 Winter 2008

Homework due – Wednesday, February 20, 2008

1. From (Chapter 9) page 181 of the Course notes (called *chapters* on the course web page), complete the following exercises:
   - Exercise 1
   - Exercise 2
   - Exercise 3
   - Exercise 4

2. Suppose you are given the values of f and f' at points x_0+h and x_0-h and you wish to approximate $f'(x_0)$. Find coefficients α and β that make the following approximation accurate to $O(h^4)$:

$$f'(x_0) \approx \alpha \frac{f'(x_0+h)+f'(x_0-h)}{2} + \beta \frac{f(x_0+h)-f(x_0-h)}{2h}$$

Compute the coefficients by combining the Taylor series expansions of $f(x)$ and $f'(x)$ about the point x_0:

$$\begin{aligned} f(x) &= f(x_0)+(x-x_0)f'(x_0)+\frac{(x-x_0)^2}{2!}f''(x_0)+\frac{(x-x_0)^3}{3!}f'''(x_0) \\ &\quad +\frac{(x-x_0)^4}{4!}f^{(4)}(x_0)+\frac{(x-x_0)^5}{5!}f^{(5)}(c_1) \end{aligned}$$

$$f'(x) = f'(x_0)+(x-x_0)f''(x_0)+\frac{(x-x_0)^2}{2!}f'''(x_0)+\frac{(x-x_0)^3}{3!}f^{(4)}(x_0)+\frac{(x-x_0)^4}{4!}f^{(5)}(c_2)$$

Hint: Combine the Taylor expansions into $(f(x_0+h)-f(x_0-h))$ and $(f'(x_0+h)+f'(x_0-h))$ and then combine these two to cancel the leading order error term (in this case $O(h^2)$).

Note: This technique for computing derivatives is useful for interpolating between points where a function and its derivative are known.