Homework due – Wednesday, February 20, 2008

1. From (Chapter 9) page 181 of the Course notes (called chapters on the course web page), complete the following exercises:
   - Exercise 1
   - Exercise 2
   - Exercise 3
   - Exercise 4

2. Suppose you are given the values of \( f \) and \( f' \) at points \( x_0 + h \) and \( x_0 - h \) and you wish to approximate \( f'(x_0) \). Find coefficients \( \alpha \) and \( \beta \) that make the following approximation accurate to \( O(h^4) \):

\[
f'(x_0) \approx \alpha \frac{f'(x_0 + h) + f'(x_0 - h)}{2} + \beta \frac{f(x_0 + h) - f(x_0 - h)}{2h}
\]

Compute the coefficients by combining the Taylor series expansions of \( f(x) \) and \( f'(x) \) about the point \( x_0 \):

\[
f(x) = f(x_0) + (x - x_0)f'(x_0) + \frac{(x - x_0)^2}{2!}f''(x_0) + \frac{(x - x_0)^3}{3!}f'''(x_0)
\]
\[
+ \frac{(x - x_0)^4}{4!}f^{(4)}(x_0) + \frac{(x - x_0)^5}{5!}f^{(5)}(c_1)
\]

\[
f'(x) = f'(x_0) + (x - x_0)f''(x_0) + \frac{(x - x_0)^2}{2!}f'''(x_0) + \frac{(x - x_0)^3}{3!}f^{(4)}(x_0) + \frac{(x - x_0)^4}{4!}f^{(5)}(c_2)
\]

**Hint:** Combine the Taylor expansions into \( (f(x_0 + h) - f(x_0 - h)) \) and \( (f'(x_0 + h) + f'(x_0 - h)) \) and then combine these two to cancel the leading order error term (in this case \( O(h^2) \)).

**Note:** This technique for computing derivatives is useful for interpolating between points where a function and its derivative are known.