Math 498 Chartier/Greenbaum
Homework #3 Winter 2008

Homework due – Wednesday, February 6, 2008

1. You can estimate the area inside the ellipse

$$\frac{x^2}{4} + y^2 = 1$$

by generating random numbers uniformly distributed in the rectangle $[-2,2] \times [-1,1]$, and seeing what fraction lie inside the ellipse. The fraction that fall inside the ellipse times the area of the rectangle (which is 8) gives an estimate of the area inside the ellipse. First, plot this ellipse. You can do this in MATLAB by typing

```
x = [-2:.01:2];
y = sqrt(1 - x.^2/4);
plot(x,y,'-r', x,-y,'-r')
hold on
```

Initialize a counter of the number of points that land inside the ellipse: `inside = 0`. Now generate 1000 random points (x_i, y_i), where x_i comes from a uniform distribution between -2 and 2 and y_i comes from a uniform distribution between -1 and 1. Plot each of these points on the same graph, using the command

```
plot(x_i,y_i,'.')
```

Test whether $x_i^2/4 + y_i^2 \leq 1$, and if so increment the counter:

```
if x_i^2/4 + y_i^2 <= 1, inside = inside+1; end;
```

At the end of your loop, determine the fraction of points that landed inside the ellipse and multiply that fraction by 8 to obtain an estimate of the area inside the ellipse. Turn in your plot and your estimate of the area inside the ellipse.

(a) Let X_i be a random variable that is 1 if the ith point lies inside the ellipse and 0 otherwise. Based on your results, estimate the mean and variance of X_i.

(b) Let $A_{1000} = \frac{1}{1000}\sum_{i=1}^{1000} X_i$. Using the Central Limit Theorem, estimate the mean, variance, and standard deviation of A_{1000}.

(c) Using the results of part (b), estimate the mean, variance, and standard deviation of $8A_{1000}$, which is the approximation to the area of the ellipse. Based on this result, how much confidence do you have in your answer? (Is it likely to be off by, say, .001, .01, .1, 1, 10, or some other amount?)

2. Do problems 1 through 4 on p. 264 of the Eigenvalues section of the Course Notes (Called *evpages* on the course web page).