Math 466, Spring 2003

Assignment 5.

Due Friday, May 23.

Objectives: To study the FFT and its use in fast Poisson solvers.

(1) Let A be an m by m TST (Toeplitz symmetric tridiagonal) matrix with α's on its main diagonal and β's on its first sub- and super-diagonal:

$$A = \begin{pmatrix} \alpha & \beta & & \\ \beta & \ddots & \ddots & \\ & \ddots & \ddots & \beta \\ & & \beta & \alpha \end{pmatrix}$$

Show that the eigenvalues and eigenvectors of A are:

$$\lambda_j = \alpha + 2\beta \cos\left(\frac{\pi j}{m+1}\right), \quad j = 1, \ldots, m,$$

$$q_\ell^{(j)} = \sqrt{\frac{2}{m+1}} \sin\left(\frac{\pi j \ell}{m+1}\right), \quad j, \ell = 1, \ldots, m.$$

(2) This is a *written* exercise. Let $\mathbf{f}$ be a vector of length 8 with components $f_0, f_1, \ldots, f_7$. Write down and clearly explain the sequence of operations you would use to compute the discrete Fourier transform of $\mathbf{f}$, using the FFT technique. Your explanation should be sufficiently clear that someone unfamiliar with the FFT could follow your instructions and compute the FFT of any given vector of length 8.