Assignment 4.

Due Wednesday, May 7 **at the start of class**. Solutions will be gone over in class and late papers will not be accepted.

Objective: To study stability and accuracy of difference methods for hyperbolic partial differential equations.

(1) Do problem 1 on p. 649 (written exercise).

(2) Do problem 4 on p. 666 (written exercise).

(3) Write a code to solve the wave equation:

$$u_{tt} = cu_{xx}, \quad 0 \leq x \leq 1, \quad t \geq 0$$

$$u(x,0) = f(x), \quad u_t(x,0) = g(x), \quad u(0,t) = u(1,t) = 0,$$

using the difference method:

$$\frac{u_i^{m+1} - 2u_i^m + u_i^{m-1}}{(\Delta t)^2} = -\frac{c}{h^2}\left(2u_i^m - u_{i+1}^m - u_{i-1}^m\right)$$

Take $f(x) = x(1-x)$, $g(x) = 0$, and $c = 2$, and go out to time $t = 2$. Plot the solution at each time step; you should see the solution behave like a plucked guitar string. Experiment with different values of h and Δt in order to verify numerically the stability condition:

$$\Delta t \leq \frac{h}{\sqrt{c}}.$$

Turn in plots of your solutions for values of h and Δt that satisfy the stability conditions and also some plots showing what happens if the stability requirements are not met.