Assignment 3.

Due Friday, April 25.

Objective: To study partial differential equations in two spatial dimensions and to write a code to solve the heat equation.

(1) Do problem 1 on p. 633.

(2) Write down a second-order accurate finite difference approximation to the Laplacian $\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2$, when the mesh spacing in the x direction is h_x and that in the y direction is h_y.

Suppose you wish to solve Poisson's equation $\Delta u = f$ on the unit square with $u = g$ on the boundary. Using a 4×3 grid (that is, 3 interior mesh points in the x direction and 2 interior mesh points in the y direction), write in matrix form the system of linear equations that you would need to solve in order to obtain a second-order accurate approximate solution.

(3) Write a code to solve the heat equation in one spatial dimension:

$$u_t = u_{xx}, \quad 0 \le x \le 1, \quad t \ge 0$$

$$u(x, 0) = u_0(x), \quad u(0, t) = u(1, t) = 0,$$

using the backward Euler method:

$$\frac{u_i^{m+1} - u_i^m}{\Delta t} = -\frac{1}{h^2}\left(2u_i^{m+1} - u_{i+1}^{m+1} - u_{i-1}^{m+1}\right)$$

Take $u_0(x) = x(1-x)$ and go out to time $t = 1$. Plot the solution at time $t = 0$, at time $t = 1$, and at various times along the way. Experiment with different mesh sizes and different timesteps, and report any observations you make about accuracy and stability.