Assignment 1.

Due Friday, Apr. 11.

Objectives: To look at existence and uniqueness of solutions to two-point boundary value problems; to practice writing out difference equations for such problems, and to solve a heat flow problem numerically.

(1) p. 588–589, problems 10 and 15.

(2) Consider the two-point boundary value problem

$$u'' + 2xu' - x^2u = x^2, \quad u(0) = 1, \quad u(1) = 0.$$

(a) Let $h = 1/4$ and explicitly write out the difference equations, using centered differences for all derivatives.

(b) Repeat part (a) using the one-sided approximation

$$u'(x_i) \approx \frac{u(x_i) - u(x_{i-1})}{h}.$$

(c) Repeat part (a) for the boundary conditions $u'(0) + u(0) = 1$, $u'(1) + \frac{1}{2}u(1) = 0$.

(3) A rod of length 1 meter has a heat source applied to it and it eventually reaches a steady-state where the temperature is not changing. The conductivity of the rod is a function of position x and is given by $c(x) = 1 + x^2$. The left end of the rod is held at a constant temperature of 1 degree. The right end of the rod is insulated so that no heat flows in or out from that end of the rod. This problem is described by the boundary value problem:

$$\frac{d}{dx}\left((1+x^2)\frac{du}{dx}\right) = f(x), \quad 0 \le x \le 1,$$

$$u(0) = 1, \quad u'(1) = 0.$$

(a) Write down a set of difference equations for this problem. Be sure to show how you do the differencing at the endpoints.

(b) Write a MATLAB code to solve the difference equations. You can test your code on a problem where you know the solution by choosing a function $u(x)$ that satisfies the boundary conditions and determining what $f(x)$ must be in order for $u(x)$ to solve the problem. Try $u(x) = (1-x)^2$. Then $f(x) = 2(3x^2 - 2x + 1)$.

(c) Try several different values for the mesh size h. Based on your results, what would you say is the order of accuracy of your method?