Homework on Linear Support Vector Machines

Due Monday, March 6.

1. Consider the following two sets of two dimensional data:

<table>
<thead>
<tr>
<th></th>
<th>x_i</th>
<th>y_i</th>
<th></th>
<th>x_i</th>
<th>y_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2</td>
<td>1.0</td>
<td></td>
<td>5</td>
<td>-9.0</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>0.0</td>
<td></td>
<td>6</td>
<td>-3.0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.5</td>
<td></td>
<td>7</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4.0</td>
<td></td>
<td>8</td>
<td>3.0</td>
</tr>
</tbody>
</table>

(a) Plot the first four data points using +’s and the second four using o’s. You can use the Matlab command:

\[
\text{plot}(x(1:4),y(1:4),'+',x(5:8),y(5:8),'o')
\]

(b) Use Matlab routine \texttt{quadprog} to find the hyperplane (i.e., the straight line) that separates the first four data points from the second four with the maximum margin. That is, solve the quadratic programming problem:

\[
\min_{w \in \mathbb{R}^2, \beta \in \mathbb{R}} \quad w^T w \quad \text{subject to:} \quad (x_i, y_i)^T w + \beta \geq 1, \quad i = 1, \ldots, 4; \quad (x_i, y_i)^T w + \beta \leq -1, \quad i = 5, \ldots, 8.
\]

After finding \(w\) and \(\beta\), plot (on the same graph with the data points) the separating hyperplane:

\[
\{(x, y) : (x, y)^T w + \beta = 0\}.
\]

(c) Now add the point \((x_9, y_9) = (1.5, 8.0)\) to the set that are plotted with o’s. Find the hyperplane that most nearly separates the two sets of data points now. That is, choose a positive constant \(C\), say \(C = 10\), and solve the quadratic programming problem:

\[
\min_{w \in \mathbb{R}^2, \beta \in \mathbb{R}} \quad w^T w + C \sum_{i=1}^{9} \epsilon_i \quad \text{subject to:} \quad (x_i, y_i)^T w + \beta \geq 1 - \epsilon_i, \quad i = 1, \ldots, 4; \quad (x_i, y_i)^T w + \beta \leq -1 + \epsilon_i, \quad i = 5, \ldots, 9, \quad \epsilon_i \geq 0, \quad i = 1, \ldots, 9.
\]

Turn in a separate plot of the 9 data points and the approximately separating hyperplane that you compute now. Print out the input arguments that you sent to routine \texttt{quadprog} and the results that it returned.