Homework on Linear Programming

Due Friday, March 9.

1. Recall the problem of life vests and life boats from Chapter 1. For simplicity consider smaller boats with the parameters:

\[
\begin{align*}
C_1 &= 1 \text{ (capacity of each vest)} \\
C_2 &= 2 \text{ (capacity of each boat)} \\
V_1 &= 1 \text{ (volume of each vest)} \\
V_2 &= 5 \text{ (volume of each boat)} \\
V &= 15 \text{ (total space available)}
\end{align*}
\]

(a) Set up the problem of maximizing the number of life boats that can be carried as a linear programming problem. **To begin with, ignore the constraints that the number of vests and the number of boats must be integers.**

(b) Use the "graphical method" to find the solution to this problem (ignoring the integer constraints). Draw the feasible region in the \(x_1 - x_2\) plane and draw contour lines of the objective function, indicating the corner of the feasible set at which the optimum occurs. Solve the appropriate linear system to find this point.

(c) Now introduce the constraint that \(x_1\) and \(x_2\) must be integers. This means that only a finite number of discrete points from the previous feasible set are still feasible (points for which \(x_1\) and \(x_2\) are both integers). Find all feasible points and plot them. Use the graphical method to solve this integer programming problem.

2. Consider Model 20 from

http://www.math.washington.edu/~burke/crs/407/models/

(a) Set this up as a linear programming problem.

(b) Solve this using Matlab, Lindo, or other linear programming software. (Note: In Matlab, you can specify the upper bounds as \(UB = [\text{Inf; Inf; 20}],\) where \(\text{Inf}\) means infinity.)

(c) Which of the constraints are "binding" at the optimum? (i.e., which of the three machines are being used at maximum capacity and which are not?)