Practice Problems on Limits

1. Use the definition of a limit to show that $\lim_{n\to\infty}(1-2^{-n})=1$. That is, given any $\epsilon>0$, find a number N such that $|(1-2^{-n})-1|<\epsilon$, whenever $n>N$.

2. Let $s_n=\frac{2n+(-1)^n}{3n}$. Use the definition of a limit to show that $\lim_{n\to\infty}s_n=\frac{2}{3}$.

3. Calculate the following limits and use the theorems on limits of sums and products and quotients and the squeezing principle or the bounded monotone sequence theorem to justify your answers. (You do not need to go back to the definition of a limit. You may use limits that have already been proved in class or on homework problems, such as $\lim_{n\to\infty}\frac{1}{n^k}=0$ for any positive integer k or $\lim_{n\to\infty}r^n=0$ if $|r|<1$.)

   (a) $\lim_{n\to\infty}\frac{n^3-1}{3n^3+n-4}$

   (b) $\lim_{n\to\infty}\frac{n\cos n}{n^2+24}$

   (c) $\lim_{n\to\infty}\frac{2^n+1}{2^n-5}$

   (d) $\lim_{n\to\infty}[(n+1)^{1/3}-n^{1/3}]$

   (e) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^2}{n^3}$ [Hint: Use the formula proved in class for $\sum_{k=1}^{n}k^2$.]

   (f) $\lim_{n\to\infty}\left(\frac{n^3}{2n^2-1}-\frac{n^2}{2n+1}\right)$

4. Show that none of the following sequences have (finite) limits. Which of these sequences are *bounded*? Which are *monotone* (increasing or decreasing)?

   (a) $n!$

   (b) $\sin\frac{n\pi}{2}$

   (c) $(-1)^n+\frac{1}{n}$

   (d) r^n if $|r|>1$

5. We have always spoken of *the* limit of a sequence as though it were impossible for a sequence to have more than one limit. Prove that this is so.