Assignment 5. Due Fri., Oct. 31.

Reading: Sec. 16.31 and 16.5.

1. p. 519–520, problems 1, 2, 3, and 4.

2. Which of the following sequences are Cauchy sequences? Give a line or so of explanation for each:

   (a) $\left|(-1)^n + \frac{1}{n}\right|$

   (b) $\sin(n)$

   (c) $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{2 + x_n}$, $n = 1, 2, \ldots$.

3. Let

$$a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}, \quad b_n = 1 - \frac{1}{1!} + \frac{1}{2!} - \ldots + \frac{(-1)^n}{n!}.$$

   (a) Show that

$$a_n \le 1 + \sum_{k=0}^{n-1} \frac{1}{2^k} < 3,$$

   for all positive integers n. Citing a theorem about bounded monotone sequences, prove that the sequence $\{a_n\}$ has a limit.

   (b) Show that for all positive integers m and n,

$$|b_m - b_n| \le |a_m - a_n|.$$

   Deduce that $\{b_n\}$ is a Cauchy sequence and therefore has a limit.