Assignment 5. Due Fri., Oct. 31.

Reading: Sec. 16.31 and 16.5.

1. p. 519–520, problems 1, 2, 3, and 4.

2. Which of the following sequences are Cauchy sequences? Give a line or so of explanation for each:
   (a) $|(-1)^n + \frac{1}{n}|$
   (b) $\sin(n)$
   (c) $x_1 = \sqrt{2}, x_{n+1} = \sqrt{2 + x_n}, n = 1, 2, \ldots$

3. Let
   $$a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}, \quad b_n = 1 - \frac{1}{1!} + \frac{1}{2!} - \ldots + \frac{(-1)^n}{n!}.$$  

   (a) Show that
   $$a_n \leq 1 + \sum_{k=0}^{n-1} \frac{1}{2^k} < 3,$$
   for all positive integers $n$. Citing a theorem about bounded monotone sequences, prove that the sequence \{a$_n$\} has a limit.

   (b) Show that for all positive integers $m$ and $n$,
   $$|b_m - b_n| \leq |a_m - a_n|.$$  

   Deduce that \{b$_n$\} is a Cauchy sequence and therefore has a limit.