Week 1 Worksheet

0. Do the mini quiz problem. Put your work and solution here:

1. Given a system of equations, is it guaranteed that the Gaussian Elimination process will eventually stop? Explain your answer. Use your answer to make some connection between systems in general and systems in echelon form.

2. (a) A system is in **reduced echelon form** if
   (i) _____,
   (ii) _____, and
   (iii) _____.
   (b) With part (a) in mind, describe how you can modify Gaussian Elimination to find an equivalent system in reduced echelon form. This is called **Gauss-Jordan elimination.**

   Step 1

   Step 2

3. Use Gauss-Jordan elimination to put the following system into reduced echelon form. Use this to solve the system.

$$\begin{aligned} y + \ z &= 3 \\ x + y - 2z &= 0 \\ x \quad + 2z &= 2 \end{aligned}$$

4. (Number of solutions; Theorem 1.3 in textbook.) **Theorem:** A system of equations has either
   (a) _____,
   (b) _____, or
   (c) _____.
   Use tools we have developed (augmented matrices, Gaussian-Elimination, echelon form) to explain why this theorem is true.

5. (GeoGebra) Consider Problem 3. Put the given three planes into GeoGebra. Then put *each* new plane that your row operations created into GeoGebra as well. Notice that row operations (a) and (b) do not change the planes, but row operation (c) does. However, what does row operation (c) keep the same? (To help answer this, just look at the two rows used in operation (c) and determine what happens to them.)

6. The points $(1, -1, 0)$, $(1, 0, 2)$, and $(0, -1, 1)$ all lie on a plane in $\mathbb{R}^3$. This plane can be written

$$ax + by + cz = d.$$

Use techniques from class to determine what a, b, c, d are.

Is there more than one possibility for a, b, c, d? Do these different values correspond to different planes?