Math 461

Friday, March 6

Chapter 12 In-class problems I

0. (a) What is a planar graph?

   (b) State Euler's theorem on connected planar graphs as an alternating sum of V, E, and F. (Where V, E, and F are the number of vertices, edges, and faces respectively.)

   (c) It's possible to draw a given planar graph G in many ways. Will any planar drawing of G have the same number of faces? Why or why not?

   (d) If we simply know the definition of planar (and not any subsequent tools), which is easier, showing that a graph is planar or showing that it's not planar? Why?

1. Is $K_{2,2,2}$ planar?

2. Let G be a connected simple planar graph with $V \geq 3$.

   (a) Prove that $3F \leq 2E$.

   (b) Prove that $E \leq 3V - 6$.

   (c) Are Parts (a) and (b) true if G is not simple?

3. (a) Does 2(b) say anything about K_5?

   (b) Does 2(b) say anything about $K_{3,3}$?

4. (a) Let G be a connected simple planar graph. Prove that G has a vertex v such that $\deg(v) \leq 5$.

   (b) Is this true if G is not simple?

5. (a) Let G be a connected simple planar graph with $V \geq 3$. Show that if G does not contain any triangles then $E \leq 2V - 4$.

   (b) Does Part (a) say anything about $K_{3,3}$?

6. (a) What does it mean for two graphs G and H to be *edge-equivalent*? (This is also called *homeomorphic*.)

   (b) Give an example of two graphs that are homeomorphic but not isomorphic. Are isomorphic graphs always homeomorphic?

   (c) **Kuratowski's Theorem.** A graph G is planar if and only if it does not contain a subgraph that is homeomorphic to K_5 and $K_{3,3}$.

   Don't prove this statement, but explain some of the implications.

7. The following is known as the Petersen graph. Graph theorists love it. Determine whether the Petersen graph is planar.