Daily problems
Assigned 8/5/19
Due 8/7/19

You must attempt each problem, ideally solving as many as possible. We will spend the first part of the next lecture discussing these problems, and you will regularly be asked to present your ideas. (You do not need to fully solve a problem to present it.)

In other words, even if you cannot completely solve a problem, you should work through some related examples or solve a simpler version of the problem for each of these problems, and you should be ready to present your ideas.

The goal of this daily assignment is to investigate Helly's Theorem when $d = 1$. The results of this problem are useful for the last problem on our weekly assignment.

(1) (a) Recall what it means for a set in $\mathbb{R}^1$ to be convex. (I.e., state all possible forms of convex sets in $\mathbb{R}^1$.)
  (b) Let $A_1, A_2, A_3 \subseteq \mathbb{R}^2$ be convex, and assume that the intersection of any two of them is nonempty. Show that the intersection of all three of them must be nonempty.
  (c) *Assume* that the following holds for any n convex sets in $\mathbb{R}^1$ (where n is some number greater than or equal to 3): If the intersection of any two of these sets is nonempty, then the intersection of all of them is nonempty. *Use this to show that* the same holds for any $n + 1$ convex sets in $\mathbb{R}^1$.
  (d) Together, what do Parts (b) and (c) show?

(2) Consider the following examples.
  (a) Let $A_n = [n, \infty)$ for each $n \in \mathbb{N}$. What is $A_i \cap A_j$ for any $i, j \in \mathbb{N}$? What is $\bigcap_{n\in\mathbb{N}} A_n$?
  (b) Let $B_n = (0, 1/n)$ for each $n \in \mathbb{N}$. What is $B_i \cap B_j$ for any $i, j \in \mathbb{N}$? What is $\bigcap_{n\in\mathbb{N}} B_n$?
  (c) What's the deal with these examples? Do they relate to Helly's Theorem? What might be going wrong here?