Daily problems
Assigned 8/16/19
Due 8/19/19

You must attempt each problem, ideally solving as many as possible. We will spend the first part of the next lecture discussing these problems, and you will regularly be asked to present your ideas. (You do not need to fully solve a problem to present it.)

In other words, even if you cannot completely solve a problem, you should work through some related examples or solve a simpler version of the problem for each of these problems, and you should be ready to present your ideas.

(1) A point $x = (x_1, x_2, \ldots, x_d) \in \mathbb{R}$ is a **lattice point** if each x_i is an integer. A polytope P is a **lattice polytope** if all its vertices are lattice points. For each of the following lattice polygons (i.e., 2-dimensional lattice polytopes), calculate

$$\begin{aligned} i &= \# \text{ of lattice points in the interior of } P \\ b &= \# \text{ of lattice points on the boundary of } P \\ A &= \text{area of } P \end{aligned}$$

(a) $P = \operatorname{conv}\{(1,1),(4,4),(5,2)\}$
(b) $P = \operatorname{conv}\{(1,3),(2,4),(3,4),(5,2),(5,1),(3,1)\}$
(c) $P = \operatorname{conv}\{(0,0),(m,0),(m,n),(0,n)\}$ (Assume $m, n \geq 1$.)
(d) $P = \operatorname{conv}\{(0,0),(0,2),(\ell,1)\}$ (Assume $\ell \geq 1$.)
Note: Your answers for (c) and (d) will vary based on m, n and ℓ.

(2) Count the number of points in the interior and on the boundary of

$$P = \operatorname{conv}\{(1,1,0),(1,0,0),(0,1,0),(0,0,1)\}$$

and

$$Q = \operatorname{conv}\{(1,1,0),(1,0,0),(0,1,0),(0,0,2)\}.$$

Do these polytopes have the same volume or different volumes? (A program like GeoGebra (free online at geogebra.org) might be helpful in answering this question. If you want to draw a 2-dimensional face of a polytope P in GeoGebra, type "Polygon($A_1, A_2, \ldots, A_n$)" where $A_1, A_2, \ldots, A_n$ are the vertices of the face.)