Daily problems
Assigned 7/26/19
Due 7/29/19

You must attempt each problem, ideally solving as many as possible. We will spend the first part of the next lecture discussing these problems, and you will regularly be asked to present your ideas. (You do not need to fully solve a problem to present it.)

In other words, even if you cannot completely solve a problem, you should work through some related examples or solve a simpler version of the problem for each of these problems, and you should be ready to present your ideas.

(1) (a) Let H be a regular hexagon (i.e. all angles and all side lengths are the same). Write H as a Minkowski sum of two other objects A and B in as many ways as possible. (I.e., find all possible A and B such that $A + B = H$.)
  (b) Do the same thing for T, where is the trapezoid with vertices $(-2, 0), (-1, 1), (1, 1), (2, 0)$.

(2) Let $A \subseteq \mathbb{R}^d$. The *complement* of A is

$$A^c = \mathbb{R}^d \setminus A = \{x \in \mathbb{R}^d \ : \ x \notin A\}.$$

  (a) Describe all A in $\mathbb{R}^1$ such that *both* A and A^c are convex.
  (b) Do the same for A in $\mathbb{R}^2$. Can you generalize this to $\mathbb{R}^d$?

(3) (a) Let $\mathcal{A}$ be a collection of convex sets in $\mathbb{R}^d$. Show that

$$B = \bigcap_{A \in \mathcal{A}} A$$

  is a convex set. (I.e., show that the intersection of convex sets is again convex.)
  (b) Give an example to show that the union of convex sets is not necessarily convex.