

MATH 113 Introduction to Abstract Algebra: Syllabus

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| Instructor: | Gabriel Dorfsman-Hopkins (gabrielhdh@berkeley.edu) |
| Homework Grader: | Kunaal Sundara (kunaalsundara@berkeley.edu) |
| GSI: | Jeremy Meza (jdmeza@berkeley.edu) |
| Lecture: | T,Th 5:00-6:30 PM (Prerecorded on Zoom) |
| Office Hours : | During scheduled class hours. or by appointment |
| Text: | Abstract Algebra, 3rd Edition, by Dummit and Foote [DF] |
| Secondary Text: | Paulin's Introduction to Abstract Algebra [P]: https://math.berkeley.edu/~apaulin/AbstractAlgebra.pdf |
| Course Website: | http://www.gabrieldorfsmanhopkins.edu/m113sp21/index.html Grades will be posted on bcourses.berkeley.edu . Homework will be collected and graded at gradescope.com . |

Objectives

You are likely familiar with algebra as being a (perhaps tedious) exercise in solving equations. The structures of addition and multiplication, and the way they intertwine, allow us explicitly to extract information (solve) from relationships (equations).

Abstract algebra is the rigorous study binary operations, that is, functions which take two inputs and one output. You are already familiar with some binary operations (addition and multiplication of integers, for example), but it turns out there are many many more (addition and multiplication of matrices, composition of functions, mixing colors, applying symmetries, permutations and card shuffles, the list goes on). In studying the abstract properties of binary operations and their interactions, we will discover that they all share many very strong underlying structural properties, which allows us to extract information for given relationships (i.e., solve equations), in many different contexts. This leads to applications in cryptology, geometry, logic and even philosophy which we may glance at if time allows.

Along the way we will gain experience in proof writing and mathematical exposition and communication, and get first hand exposure to the abstract axiomatic approach pervasive within all branches of theoretical mathematics.

Structure

Due to the ongoing COVID-19 pandemic, the course will be entirely remote and asynchronous. I will post 2 recorded lectures a week, corresponding to our scheduled Tuesday, Thursday lectures, but they will be available to watch at your convenience. I will record them on Zoom and post a link on the course website.

That being said, Mathematics is not a spectator sport, and watching someone do abstract algebra is much like reading the New York Times backwards. All the information may be there, but it will take some unscrambling to make sense of it. For this reason I will have exercises embedded in the lectures where I will suggest you pause the video and work out some of the details. There will also be many written exercises collected as homework assignments, or takehome exams.

I will be available hosting zoom office hours during the usual course hours (5-6:30 on Tuesdays and Thursdays), so if you are watching at that time feel free to drop in and ask any questions you have. The GSI will also be hosting office hours where you can go ask questions. The times of GSI office hours are currently TBA but I will keep you posted.

Discord

I will be curating a channel on discord, and will send out invitations over email. There you will be able to ask questions about homework and lectures, as well as have informal meetings on an audio/video channel. I will monitor the channel and will try to answer questions promptly, but you should feel free to answer questions posted there as well. I hope this is a space for group study and discussion, and to serve as a sort of *open office door* during remote learning.

Homework

There will be homework collected almost every week **on Fridays at Noon**. They will be assigned the week before they are due. These assignments will be proofs, as well as computations and explorations of examples. It is preferable that they are typed up using L^AT_EX, or a similar mathematical typesetting language. Feel free to work in groups, but each student must write up their results separately.

Take Home Tests

There will be four takehome tests. These will look similar to the homework assignments but differ in the following important ways. First, they will be assigned on Friday and due the following Monday **at 5 PM** (except the final which will be available during finals week), and subsequently will be a bit shorter (the corresponding homework assignments those weeks will be shorter as well). Second, you must work on them yourself. You may use the course texts ([DF] and [P]), as well as your course notes. You may *not* use the internet or your peers. They are in theory cumulative, but in practice will reflect material most recently covered.

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| Takehome Test 1: | Assigned Friday 2/19 | Due Monday 2/22 |
| Takehome Test 2: | Assigned Friday 3/19 | Due Monday 3/22 |
| Takehome Test 3: | Assigned Friday 4/23 | Due Monday 4/26 |
| Takehome Test 4: | Assigned Monday 5/10 | Due Friday 5/14 |

Grading

Raw grades will be computed* as follows:

| <u>Category</u> | <u>Percentage</u> |
|-----------------|-------------------|
| Homework | 40% |
| Takehome 1 | 15% |
| Takehome 2 | 15% |
| Takehome 3 | 15% |
| Takehome 4 | 15% |
| Total: | 100% |

*Loosely speaking, the A range corresponds to scores in the 90s, the B range to scores in the 80s, the C range to scores in the 70s, and the D range to scores in the 60s. That being said, after evaluating the performance of the class over the entire quarter, I may adjust letter grades according to the median raw score.

Make-ups and Extensions

If you need an extension on homework or the projects, let me know *as soon as possible*. I like to post solutions ASAP after assignments are turned in, and once they are posted I will no longer accept late assignments. I do understand that we are in the midst of a global pandemic, and things are unpredictable, so communicate with me early and often if you need more flexibility. I will be more flexible with homework assignments than I will with the takehome exams.

Disabled Students' Program (DSP)

The University of California is committed to providing access, equal opportunity and reasonable accommodation in its services, programs, activities, education and employment for individuals with disabilities. These resources include exam proctoring and accommodations in distraction free environments and with extra time as well as note taking. To request disability accommodation contact the DSP Office at least ten days in advance at (510) 642-0518(V), (510) 642-6376(TTY), (510) 643-9686(FAX), or dsp@berkeley.edu.

If you have a letter from the Disabled Students Program (DSP) indicating that you have a disability which requires academic accommodations, please present the letter to me **as soon as possible** so we can discuss the accommodations you need.

COVID-19 Addendum

We are in the midst of a global pandemic, and everything is unpredictable. As such, things might change rapidly, both in the structure of the course and how we respond to it, and we must be ready to adapt. I have a long term sub lined up in case I get sick, or have a family member I must take care of. If you or your family were to become ill don't hesitate to contact me and we will work something out to keep you from falling too behind, or being penalized. It is most important to stay safe and healthy and prevent the spread.

Course Schedule

On the next page is a *very rough* schedule of the course, organized weekly. It is very difficult to predict pace for an online course during a pandemic. We may go faster than is laid out over the first few weeks...or slower. We may cover all these things and more, or we may skip some sections due to time. Since projects are specific implementations, their due dates may shift due to timing issues.

- Week 1 (1/19-1/22):** Introduction. Sets, functions, proofs. [P] Chapter 1 and [DF] 0.1
- Week 2 (1/25-1/29):** Properties of the integers. [P] Chapter 2, [DF] 0.2-0.3.
Definition, examples, and early properties of groups. [DF] 1.1-1.5.
Homework 1 due Friday.
- Week 3 (2/1-2/5):** Group homomorphisms and group actions. [DF] 1.6-1.7
Subgroups, properties, and examples. [DF] 2.1-2.3.
Homework 2 due Friday.
- Week 4 (2/8-2/12):** Subgroups and generators. [DF] 2.3-2.5.
Homomorphisms and quotients of groups. [DF] 3.1-3.2.
Homework 3 due Friday.
- Week 5 (2/15-2/19):** The isomorphism theorems. [DF] 3.2-3.5.
Homework 4 due Friday
Takehome 1 assigned Friday
- Week 6 (2/22-2/26):** Group Actions. [DF] 4.1-4.4.
Takehome 1 due Monday
Homework 5 due Friday
- Week 7 (3/1-3/5):** Sylow theorems. [DF] 4.5-4.6.
Direct products. [DF] 5.1-5.3.
Homework 6 due Friday
- Week 8 (3/8-3/12):** Semidirect products. [DF] 5.4-5.5.
Extra topics in group theory. [DF] 6.1-6.2.
Homework 7 due Friday.
- Week 9 (3/15-3/19):** Free groups. [DF] 6.3.
Homework 8 due Friday.
Takehome 2 Assigned Friday.
- Week 10 (3/22-3/26):** **Takehome 2 Due Monday**
Spring Break
- Week 11 (3/29-4/2):** Rings, first properties, homomorphisms, ideals. [DF] 7.1-7.4.
No homework collected.
- Week 12 (4/5-4/9):** Rings of fractions and the Chinese Remainder Theorem. [DF] 7.5-7.6.
Hierarchy of particularly nice rings. [DF] 8.1-8.3.
Homework 9 due Friday.
- Week 13 (4/12-4/16):** Principle Ideal Domains. [DF] 8.3.
Polynomial rings. [DF] 9.1-9.3.
Homework 10 due Friday.
- Week 14 (4/19-4/23):** More on polynomial rings. [DF] 9.4-9.5.
Introducing fields. [DF] 13.1.
Homeowork 11 due Friday.
Takehome 3 assigned Friday.
- Week 15 (4/26-4/30):** Field extensions and algebraic closures. [DF] 13.1-13.2, 13.4.
Takehome 3 due Monday
Homework 12 due Friday
Last day of formal instruction on Friday
- Week 16 (5/3-5/7):** *Review Week*
Homework 13 due Friday
- Week 17 (5/10-5/14):** **Takehome 4 Assigned Monday.**
Takehome 4 Due Friday.