Math 425
Assignment 6

due Wednesday, February 24

1. Problem 13.2.

2. Problem 13.3.

3. This is a continuation of Problem 2, Assignment 1. Let $f(x) = x^2 \sin(1/x) + \frac{1}{2}x$. From the results of that problem, show that $f'(0) \neq 0$ but that f is not invertible in any neighborhood of 0. Why does this not contradict the inverse function theorem?

4. Let $(u, v) = \mathbf{f}(x, y) = (x^2,\, y/x)$. (The domain of $\mathbf{f}$ is $\{(x, y) \in \mathbb{R}^2 : x \neq 0\}$.)

   a. What is the range of $\mathbf{f}$? Sketch some of the inverse images of the lines $u =$ constant and $v =$ constant in the xy-plane (i.e., the curves $x^2 =$ constant and $y/x =$ constant). Also sketch the images of some of the lines $x =$ constant and $y =$ constant in the uv-plane.
   b. Compute the Jacobian matrix $\mathbf{Df}(x, y)$ and its determinant $J_{\mathbf{f}}(x, y)$.
   c. Compute the local inverses of $\mathbf{f}$. What are their domains?

5. Let $(u, v) = \mathbf{f}(x, y) = (x^2 + 2xy + y^2,\, 2x + 2y)$.

   a. Compute the Jacobian matrix $\mathbf{Df}(x, y)$ and its determinant $J_{\mathbf{f}}(x, y)$.
   b. Your answer to (a) should suggest that $\mathbf{f}$ is highly noninvertible. What is the range of $\mathbf{f}$? What is the inverse image of a point (u, v) in this range?

6. Let $(u, v) = \mathbf{f}(x, y) = (x - y,\, xy)$.

   a. Compute the Jacobian matrix $\mathbf{Df}(x, y)$ and its determinant $J_{\mathbf{f}}(x, y)$.
   b. Sketch the inverse images of some of the lines $u =$ constant and $v =$ constant in the xy-plane (i.e., the curves $x - y =$ constant and $xy =$ constant). There's something unusual about the relation between these curves at the points where $J_{\mathbf{f}} = 0$: what is it?
   c. Observe that $\mathbf{f}(2, -3) = (5, -6)$. Compute explicitly the local inverse $\mathbf{g}$ of $\mathbf{f}$ such that $\mathbf{g}(5, -6) = (2, -3)$. Also compute $\mathbf{Dg}$.
   d. Show by explicit calculation that the matrices $\mathbf{Df}(2, -3)$ and $\mathbf{Dg}(5, -6)$ are inverses of each other.