From Rudin, pp. 98–99: 2, 3, 4, 7.

Problem 7 is kind of a mouthful, and the results in it are perhaps more interesting than the proofs, so here are a few thoughts to get you started. That $f$ is bounded follows from the inequality $ab \leq \frac{1}{2}(a^2 + b^2)$, valid for all real $a, b$ (because $a^2 + b^2 - 2ab = (a - b)^2 \geq 0$).

To show that $g$ is unbounded and $f$ is discontinuous at the origin, consider their behavior on the curves $x = y^3$ and $x = cy^2$ respectively. And for the last assertion it is enough to consider lines through the origin, as $f$ and $g$ are clearly continuous elsewhere.

Also the following:

1. Prove the converse of Rudin’s problem 2. That is, suppose $f : X \to Y$ is a map between metric spaces such that $f(E) \subset f(E)$ for all $E \subset X$. Show that $f$ is continuous.

2. Again suppose $f : X \to Y$ is a map between metric spaces. Show that $f$ is continuous if and only if $f^{-1}(E^\circ) \subset [f^{-1}(E)]^\circ$ for every $E \subset Y$. (Recall that $E^\circ$ is the interior of $E$.)

3. Suppose $X$ is a metric space and $f : X \to \mathbb{R}$ is continuous. Show that if $f(x_0) > 0$, there is a neighborhood $U$ of $x_0$ such that $f(x) > 0$ for all $x \in U$. 