Convergence of series Practice problems

Decide whether the following series are convergent or divergent , and give a proof.

1. $\sum_{i=1}^{\infty} \frac{3^i}{4^i+4}$
2. $\sum_{i=1}^{\infty} \frac{3^i+1}{4^i-3}$
3. $\sum_{i=1}^{\infty} \frac{3^i\, i}{4^i}$
4. $\sum_{i=1}^{\infty} \frac{3^i}{4^i\, i}$
5. $\sum_{i=1}^{\infty} \frac{i!\,(i+1)!}{(3i)!}$

Decide whether the following series are absolutely convergent, convergent but not absolutely or divergent and give a proof

6. $\sum_{i=1}^{\infty}(-1)^i \frac{3^i\, i}{4^i}$
7. $\sum_{i=1}^{\infty} \sin(i) \frac{3^i\, i}{4^i}$
8. $\sum_{i=1}^{\infty}(-1)^i \cos(\frac{1}{i})$
9. $\sum_{i=1}^{\infty}(-1)^i \sin(\frac{1}{i})$ Hint $\lim_{i\to\infty} \frac{\sin(\frac{1}{i})}{\frac{1}{i}} = 1$.
10. $\sum_{i=1}^{\infty}(-1)^i \frac{1}{\sqrt{i^2+1}}$
11. $\sum_{i=1}^{\infty}(-1)^{3i} \frac{1}{\sqrt{i^2+1}}$

Prove that if $\sum_{i=1}^{\infty} a_i$ converges and $a_i \geq 0$, and b_i is defined by $\begin{cases} b_i = 0 & \text{if i is even} \\ b_i = a_i & \text{if i is odd} \end{cases}$

then $\sum_{i=1}^{\infty} b_i$ converges also.
Is the result still true if is do not assume $a_i \geq 0$? Justify your answer.