MATH 327
Midterm Exam
Spring 2014

Name ____

Student ID #____________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:____

1	8	
2	16	
3	18	
4	8	
Total	50	

- Your exam should consist of 4 problems on 5 pages. Check that you have a complete exam.
- You are allowed to use the axioms and elementary properties of real numbers and the list of results passed out with this exam. You are not allowed to use any other sources.
- In your proofs, you may use any item on the list of results, including basic algebra that follows from the ordered field axioms for the real numbers. All other claims should be justified.
- If you need more room, use the back of the page. Indicate to the grader that you have done so. DO NOT USE SCRATCH PAPER.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (8 points)

   (a) Complete the definition.

      i. A sequence $\{a_n\}$ *converges* to a real number a if ...

      ii. A sequence $\{a_n\}$ is *Cauchy* if...

   (b) State the Cauchy Criterion for Convergence.

   (c) Give an example of a sequence that is not Cauchy. Justify your answer.

2. (16 points) State whether each of the following is TRUE or FALSE. Justify your answer by giving a proof or a counterexample.

   (a) _______ If $\{a_n\}$ is strictly increasing, then it cannot have a convergent subsequence. (If it is useful, you may use without proof the fact that every subsequence of a convergent sequence converges.)

   (b) _______ If $\{a_n\}$ is an increasing sequence of **negative** terms and $\{b_n\}$ is a decreasing sequence of **non-negative** terms, then the sequence $\{a_n b_n\}$ is monotone.

   (c) _______ If $n \in \mathbb{N}$ and a and b are real numbers such that $a \geq b \geq 0$, then

   $$a^n - b^n \geq nb^{n-1}(a-b).$$

   (d) _______ For every pair of real numbers x and y such that $x < y$, there are at least three rational numbers in the interval (x, y).

3. (18 points) Define a sequence $\{a_n\}$ by

$$a_n = \frac{n-1}{4n+1} \text{ for each } n \in \mathbb{N}.$$

(a) The sequence $\{a_n\}$ converges. Find its limit and use the Limit Properties (Sum, Product, Quotient) to prove you are correct.

(b) Give an ϵ-N verification of the limit in part (a). (Use only basic algebra, the definition of convergence, and the Archimedean Property in your proof.)

(c) Is $\{a_n\}$ monotone? Justify your answer.

THIS PROBLEM CONTINUES ON THE NEXT PAGE.

CONTINUED FROM THE PREVIOUS PAGE

(d) Let $S = \left\{ \dfrac{n-1}{4n+1} : n \in \mathbb{N} \right\}$

i. Find $\sup(S)$ and $\inf(S)$ and prove you are correct.

ii. Does S have a maximum? Justify your answer.

4. (8 points) Prove Theorem 2 from the list of results: If $\{a_n\}$ converges, then $\{a_n\}$ is bounded.