Math 327, Section B, Midterm I, February 1, 2016

Name_____

1. Use the $\epsilon - N$ definition of convergence to show that the sequence $\left(\dfrac{2n-5}{n+1} \right)_{n \in \mathbf{N}}$ converges. First, you have to find the limit to use in your proof.

2. Give examples of the following. If there is no such sequence or set, explain why not.

   (a) A decreasing, but convergent sequence.

   (b) A bounded and increasing sequence.

   (c) A closed, but not compact set.

   (d) A compact, but not closed set.

   (e) A monotone, bounded, divergent sequence.

   (f) A divergent sequence with a convergent subsequence.

3. Let $S = \left\{(-1)^n + \frac{1}{n} : n \in \mathbf{N}\right\}$. Find the supremum and infimum of the set S. Is S closed? Is S compact? Prove your claims.

4. Prove that the limit of a sequence is unique, i.e. you cannot have $a_n \to a$ and $a_n \to b$ with $a \neq b$.