FINAL

Math 327A

name

You must show all work for full credit. Use the backs of the test pages as necessary.

1. Find the limit of the sequence $\{x_n\}$ defined inductively by $x_1 = \sqrt{2}, x_{n+1} = \sqrt{2}^{x_n}$. Justify your answer. You may assume that $x_n < \sqrt{2}^{x_n}$ for any term x_n of this sequence.

2. Use the *definition* of limit to compute the limit of $n/(n^2+1)$ as $n \to \infty$. Justify your answer.

3. Let $\{x_n\}$ be a sequence of real numbers converging to $\pi/2$. Show that $\{\cos x_n\}$ converges to 0.

4. The following are *incorrect* statements of theorems discussed in class. In each case give the *correct* statement of the theorem.

a. If $f_n(x) \to f(x)$ on $[a, b]$ and f_n, f are continuous, then the convergence is uniform.

b. If $\{f_n(x)\}$ converges uniformly to the function $f(x)$ on $[a, \infty)$ and the f_n are continuous, then $\{\int_a^\infty f_n(x)dx\}$ converges to $\int_a^\infty f(x)dx$.

c. If $f_n(x) \leq a_n$ on $[a, b]$ and a_n converges to 0, then $f_n(x)$ converges to 0 uniformly.

5. Write down a power series which converges exactly for $|x| < 2$ and compute its sum explicitly.

6. Work out a power series expansion of $\tan^{-1}(x^2)$ valid for $|x| < 1$.

7. Decide whether the following sequences $f_n(x)$ of functions converge uniformly to the given function $f(x)$ on the given interval.

a. $f_n(x) = n/(x+n)$, $f(x) = 1$, on $[1, \infty)$.

b. $f_n(x) = (\tan^{-1} nx)/n^2$, $f(x) = 0$, on $[0, \infty)$.

c. $f_n(x) = \sum_{i=0}^{n} x^i$, $f(x) = 1/(1-x)$, on $[-3/4, 1/2]$

8. Show that the function $f(x) = x^3 + x^2$, regarded as defined on the closed interval $[1, 2]$, has a continuous inverse defined on the interval $[2, 12]$. You may use basic information from differential calculus.