**MIDTERM #2**

**Math 327A**

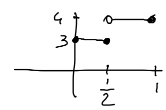
name

You must show all work for full credit. Use the backs of the test pages as necessary.

1. Give an example of a function $f$ defined on the interval $[0, 1]$ which is discontinuous at just one point in this interval, such that $f$ does *not* take on every value between $f(0)$ and $f(1)$.

$f \quad [0,1] \to \mathbb{R}$

$f(x) = \begin{cases} 3 & \text{if } x \leq \frac{1}{2} \\ 4 & \text{if } x > \frac{1}{2} \end{cases}$ discontinuous at $1/2$




$f$ does not take all values between 3 and 4

2. The following three assertion are *incorrect* statements of theorems discussed in class. In each case, give the *correct* statement of the theorem.

a. Every continuous function defined on an closed and bounded interval is uniformly continuous.

b. Every uniformly continuous function sends Cauchy sequences to Cauchy sequences.

c. A positive series $\sum_{i=1}^{\infty} a_i$ converges if and only if its partial sums are bounded.

$\sum_{n=1}^{\infty} (-1)^n$ is an example of a non convergent series but its partial sums are bounded so in general series converges $\Rightarrow$ partial sums are bounded, but not vice-verse

3. Find the pointwise limit $f(x)$ of the sequence $f_n(x) = nx/(1 + nx)$ on the closed interval $[0, 1]$. Use the formula for $f(x)$ to decide whether or not the $f_n$ converge to $f$ uniformly on this interval.

$f \quad [0,1] \to \mathbb{R}$

$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{o.w.} \end{cases}$

$f$ is not continuous at 0, so convergence is not uniform

4. Give an example of a function on the interval $[0, 1]$ that is not continuous at *any* point in this interval.

$f\ [0,1] \to R$

$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

if $x_0$ is rational $f(x_0) = 1$ but $x_n = x_0 + \frac{\sqrt{2}}{n}$ (or $x_0 - \frac{\sqrt{2}}{n}$ if $x_0 = 1$)
is irrational and $x_n \in [0\ 1]$ if $n$ is big enough
$f(x_n) = 0$ (for $n$ big enough) $\to 0 \neq f(x_0) = 1$
if $x_0$ is irrational we can always find a rational number $x_n \in [0,1]$
in the interval $(x_0 - \frac{1}{n}\ \ x_0 + \frac{1}{n})$ (because $Q$ is dense in $R$) and
$1 = f(x_n) \to 1 \neq f(x_0) = 0$

5. Show that there is a number $x$ in the interval $[0, 1]$ such that $\cos x = x$.

consider the function $f\ [0,1] \to R$
$f(x) = \cos x - x$

$f$ is continuous because it is the difference of continuous functions.
$f(0) = 1 > 0$
$f(1) = \cos(1) - 1 < 0$
by the intermediate value th. there must be $c \in [0,1]$
s.t $f(c) = 0$ so $\cos(c) - c = 0$ or $\cos(c) = c$