MIDTERM #2

Math 327A

name

You must show all work for full credit. Use the backs of the test pages as necessary.

1. Give an example of a function f defined on the interval $[0,1]$ which is discontinuous at just one point in this interval, such that f does *not* take on every value between $f(0)$ and $f(1)$.

2. The following three assertion are *incorrect* statements of theorems discussed in class. In each case, give the *correct* statement of the theorem.

a. Every continuous function defined on an interval is uniformly continuous.

b. Every continuous function sends Cauchy sequences to Cauchy sequences.

c. A series $\sum_{i=1}^{\infty} a_i$ converges if and only if its partial sums are bounded.

3. Find the pointwise limit $f(x)$ of the sequence $f_n(x) = nx/(1+nx)$ on the closed interval $[0,1]$. Use the formula for $f(x)$ to decide whether or not the f_n converge to f uniformly on this interval.

4. Give an example of a function on the interval $[0, 1]$ that is not continuous at *any* point in this interval.

5. Show that there is a number x in the interval $[0, 1]$ such that $\cos x = x$.