Math 327A

name

You must show all work for full credit. Use the backs of the test pages as necessary.

1. Show carefully, using the definition of limit, that the sequence $\{s_n\}$ defined by $s_n = \sin n / \sqrt{n}$ converges. What is its limit?

2. The following three assertion are *incorrect* statements of theorems discussed in class. In each case, give the *correct* statement of the theorem.
    a. Every increasing or decreasing sequence is convergent.
    b. Every sequence has a convergent subsequence.
    c. Every set of real numbers that is bounded below has a smallest lower bound.

3. Give three examples of sequences of real numbers, one with only one number occurring as the limit of a subsequence, another with two such numbers, and the last one with three such numbers.

4. Define a sequence $\{s_n\}$ of real numbers inductively via $s_1 = 1$, $s_{n+1} = \sqrt{s_n + 2}$. Show that this sequence converges and evaluate its limit.

5. Let $\{s_n\}$ be a convergent sequence of real numbers such that $0 \le s_n \le 1$ for all n. Show that $\lim_{n \to \infty} s_n$ lies between 0 and 1.