MATH 327 A
Final Exam
Spring 2014

Name ________________________________________

Student ID #____________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:__________________________________

| | | |
|---|---|---|
| 1 | 9 | |
| 2 | 15 | |
| 3 | 15 | |
| 4 | 11 | |
| 5 | 15 | |
| Total | 65 | |

- Your exam should consist of 5 problems. Check that you have a complete exam.
- You are allowed to use the list of results passed out with this exam. You are not allowed to use any other sources (calculator, books, notes, the internet, other people, etc).
- In your proofs, you may use any item on the list of results. All other claims should be justified.
- If you need more room, use the back of the page. Indicate to the grader that you have done so. DO NOT USE SCRATCH PAPER.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (9 points) Complete the definition.

   (a) Suppose $f_n : D \to \mathbb{R}$ for each $n \in \mathbb{N}$ and $f : D \to \mathbb{R}$. The sequence of functions $\{f_n\}$ *converges uniformly* to $f$ on $D$ if...

   (b) Suppose $D$ is a subset of $\mathbb{R}$. A number $x_0$ is a *limit point* of $D$ if...

   (c) Suppose $f : D \to \mathbb{R}$ and $x_0$ is a limit point of $D$. We say $\lim_{x \to x_0} f(x) = L$ if ...

2. (15 points) Determine whether the series converges absolutely, converges conditionally or diverges. Justify your answer.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{2^{k+1} \cdot k^2}{3^k}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{4k^{1/5}}{4k-3}$

THIS PROBLEM CONTINUES ON THE NEXT PAGE.

(c) $\displaystyle\sum_{k=1}^{\infty}(-1)^k \cdot \frac{k+1}{k^2}$

3. (15 points) For each $n \in \mathbb{N}$, define $f : [0,1] \to \mathbb{R}$ by

$$f_n(x) = \frac{x^n}{1 + x^n}.$$

(a) Determine the function $f$ to which $\{f_n\}$ converges pointwise on $[0,1]$

(b) Is convergence uniform on $[0,1]$? Prove you are correct.

(c) Suppose $0 < r < 1$. Is convergence uniform on $[0,r]$? Prove you are correct.

4. (11 points) Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} \frac{1}{x-1} & \text{if } x < 0 \\ 5 & \text{if } x = 0 \\ 7x - 1 & \text{if } x > 0. \end{cases}$$

Determine $\lim_{x \to 0} f(x)$ and use the $\epsilon$-$\delta$ definition of the limit to prove you are correct.

5. (15 points) Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \frac{x + |x|}{2}.$$

Determine the values of $x$ at which $f$ is continuous. Prove you are correct.