

Math 327 Fall 2016 Midterm 2 Exam

*Write clearly and legibly. Justify all your answers.*

*You will be graded for correctness and clarity of your solutions.*

*You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides.*

*You may use a calculator.*

*You can use elementary algebra and any result that we proved in class (but not in the homework). You need to prove everything else.*

*Please raise your hand and ask a question if anything is not clear.*

*This exam contains 7 pages and is worth a total of 50 points.*

*You have 50 minutes. Good luck*

NAME:-----

PROBLEM 1 -----

PROBLEM 2 -----

PROBLEM 3 -----

Total (50 points) -----

- **Problem 1** (10 points) Prove that if  $\sum_{i=1}^{\infty} a_i$  converges to  $a$  and  $\sum_{i=1}^{\infty} b_i$  converges to  $b$  then  $\sum_{i=1}^{\infty} (ca_i + db_i)$  converges to  $ca + db$

(5 points) Give an example of series  $\sum_{i=1}^{\infty} a_i$  and  $\sum_{i=1}^{\infty} b_i$  that both diverge and numbers  $c \neq 0$  and  $d \neq 0$  such that  $\sum_{i=1}^{\infty} (ca_i + db_i)$  converges

- **Problem 2** (20 points) Decide whether the following series are convergent or divergent and give a proof.

$$\sum_{i=1}^{\infty} \frac{i+1}{\sqrt{i^4+3}}$$

b)  $\sum_{i=1}^{\infty} \frac{i 2^i}{5^i}$

(PROBLEM 2 CONTINUED)

c)  $\sum_{i=1}^{\infty} \frac{2-(-1)^i}{i^2}$

d)  $\sum_{i=1}^{\infty} a_i$ , where  $\begin{cases} a_i = 0 & \text{if } i \text{ is even} \\ a_i = \frac{i-1}{i+1} & \text{if } i \text{ is odd} \end{cases}$

- **Problem 3** (15 points) Decide whether the following series are absolutely convergent, convergent but not absolutely or divergent and give a proof.

a)  $\sum_{i=1}^{\infty} (-1)^i \frac{2^i}{i!}$

b)  $\sum_{i=1}^{\infty} (-1)^i \frac{1}{i+5}$

(PROBLEM 3 CONTINUED)

c)  $\sum_{i=1}^{\infty} (-2)^i \frac{1}{3^{i+1}}$