Math 327 Fall 2016 Midterm 1

Write clearly and legibly. Justify all your answers.

You will be graded for correctness and clarity of your solutions.

You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides. You may use a calculator.

You can use elementary algebra and any result that we proved in class (but not in the homework). You need to prove everything else.

Please raise your hand and ask a question if anything is not clear.

This exam contains 6 pages and is worth a total of 50 points. You have 50 minutes. Good luck

NAME:_________

PROBLEM 1 (10 points) _______

PROBLEM 2 (17 points)_______

PROBLEM 3 (11 points) ________

PROBLEM 4 (12 points) ________

Total ____________

- **Problem 1** (10 points) Find $\lim_{n\to\infty} \frac{n+1}{2n+1}$ and prove your result.

- **Problem 2** Given $S = \{\frac{n+1}{2n+1} | n \in N\}$ Let $s = \sup S$ and $i = \inf S$

a) (6 points) Find the value of i and prove $i = \inf S$.

b) (6 points) Find the value of s and prove $s = \sup S$

c) (3 points) Is S closed ? Justify your answer.

d)(2 points) Is S sequentially compact ? Justify your answer.

- **Problem 3**(7 points)Prove that if the sequence $\{a_n\}$ converges to a then the sequence $\{|a_n|\}$ converges to $|a|$.(you can use the inequality ($|x-y| \geq ||x|-|y||$)

(4 points) Is it true that if $\{|a_n|\}$ converges to $|a|$ then $\{a_n\}$ converges to a ? Justify your answer.

- **Problem 4**(12 points) Say if each of the statements below is True or False (just write T or F next to each of them), and briefly explain why.

  1. A convergent sequence must be monotone and bounded.

  2. A decreasing sequence converges.

  3. A set S can have a maximum but no least upper bound.

  4. Q is closed.

  5. Q is open.

  6. Assume $\{a_n\}$ converges , and $\{b_n\}$ is a sequence. Then $\{a_n \cdot b_n\}$ converges if and only if $\{b_n\}$ converges.