Math 327 Fall 2016 Final Exam

Write clearly and legibly. Justify all your answers.

You will be graded for correctness and clarity of your solutions.

You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides. You may use a calculator.

You can use elementary algebra and any result that we proved in class (but not in the homework). You need to prove everything else.

Please raise your hand and ask a question if anything is not clear.

This exam contains 8 pages and is worth a total of 90 points. You have 1 hour and 50 minutes. Good luck

NAME:_________

PROBLEM 1 _______

PROBLEM 2 _______

PROBLEM 3 ________

PROBLEM 4 ________

PROBLEM 5________

PROBLEM 6 ________

Total ____________

- **Problem 1** (10 points) Prove that if $|r| < 1$ then $\{r^n\}$ converges (do a full proof using the definition of limit of a sequence, do not just quote a result from class)

- **Problem 2** (15 points) Decide if the following functions are uniformily continuous, and prove your answer.

  1. $f : (0, +\infty) \to R, \quad f(x) = \frac{1}{x^2}$

2. $f : (\frac{1}{2}, 2) \to R, \quad f(x) = \frac{1}{x^2}$

3. **Problem 3** (15 points) Prove that the image of a closed and bounded set under a continuous function is closed. That is, if $f : D \to R$ is continuous and D is closed and bounded, then$f(D)$ is closed.

4. **Problem 4** (15 points) Given $f\,D \rightarrow R$ define what it means to say that $\lim_{x\rightarrow x_0} f(x) = l$. We have given two equivalent definitions, you can use either one of the two.

Given $f\,D \rightarrow R$ define what it means to say the f is continuous at $x_0 \in D$.

Using the definition above, prove that $f : R \rightarrow R$, f(x)=2x+1 is continuous everywhere (do a full proof, do not just quote the result from class that sums and products of continuous functions are continuous. You can use limit laws , if you like).

5. **Problem 5** (15 points) Say whether the following series diverge or converge and justify your answer.

a) $\sum_{i=1}^{\infty} \frac{\sqrt{i^2+1}}{i^4+5}$

b) $\sum_{i=1}^{\infty} \frac{10^i}{i!}$

c) $\sum_{i=1}^{\infty} a_i$ where $\begin{cases} a_i = \frac{1}{i} & \text{if i is odd} \\ a_i = \frac{1}{i^2} & \text{if i is even} \end{cases}$

6. **Problem 6** (20 points) Consider the sequence of functions $\{f_n\}$, with $f_n : R \to R$ defined by $f_n(x) = \frac{x^{2n}}{1+x^{2n}}$

a) find the function f the sequence converges pointwise

b) does $\{f_n\}$ converge to f uniformily ? Justify your answer

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c) If you change the domain of f_n from R to $(0, \frac{1}{2})$, find the function f the sequence converges pointwise .

d) Does the sequence in part c) converge uniformily ? Justify your answer.