

Hw 8

Read chapter 9.2 and 9.3 of the textbook.

Main skills:

- You need to know the definition of pointwise and uniform convergence of functions

Do the following problems:

1. Define $f_n[0, 1] \rightarrow R$ by $f_n(x) = \frac{1}{nx+1}$
 - Find the function f the sequence $\{f_n\}$ converges to.
 - Prove the convergence is pointwise and not uniform.
 - Is the convergence uniform on $D = (0, 1)$?
2. Define $f_n[, 0, 1] \rightarrow R$ by $f_n(x) = \frac{x}{nx+1}$
 - Find the function f the sequence $\{f_n\}$ converges to.
 - Prove the convergence is uniform.
3. Consider the sequence of functions $f_k(x) = \sum_{n=0}^k \frac{x^2}{(1+x^2)^n}$ defined on $D = [0, \infty)$
 - (a) Find the function f the sequence $\{f_k\}$ converges to. Give a simple formula for f , do not write it as a series.
 - (b) Is the convergence uniform ?