Hw 7

Read chapter 3 of the textbook.

Main skills:

- You need to know the extreme value and the intermediate value theorem.
- You need to know the definition of uniform continuity

Do the following problems:

1. Find the image of the following functions :

   (a) $f[1\,,\,2] \cup [3\,,\,4] \to R \quad f(x) = x + 2$

   (b) $f[1\,,\,2] \cup [5\,,\,7] \to R$

   $$\begin{cases} x & \text{if } 1 \leq x \leq 6 \\ x + 5 & \text{if } 6 < x \leq 7 \end{cases}$$

   (c) $f[2\,,\,\infty] \to R \quad f(x) = \frac{1}{x}$

2. Prove that a continuous function f defined on a closed and bounded interval has minimum.

3. Give an example of a continuous function f that has no minimum nor maximum value.

4. Decide if the following statements are True or False, and justify your answer.

   (a) Every function $f[1\,,\,2] \to R$ has maximum

   (b) Every continuous function $f[1\,,\,2] \to R$ has maximum

   (c) Every continuous function $f(1\,,\,2) \to R$ has maximum

   (d) Every bounded function $f[1\,,\,2] \to R$ is continuous

   (e) Every continuous function with bounded image $f(1\,,\,2) \to R$ has maximum

   (f) If $f[1\,,\,2] \to R$ is continuous , then its image is an interval.

   (g) If $f[1\,,\,2] \cup [3\,,\,4] \to R$ is continuous , then its image is closed.

   (h) If $f[1\,,\,2] \cup [3\,,\,4] \to R$ is continuous , then its image is bounded

   (i) If $f[1\,,\,2] \cup [3\,,\,4] \to R$ is continuous , then its image is an interval.

5. Prove that the equation $x\sin(x) = \frac{1}{2}$ has at least one real solution.

6. Give examples of a continuous function $f\,D \to R$ such that

   (a) D is bounded but $f(D)$ is not bounded

   (b) D is closed but $f(D)$ is not closed

(c) D is bounded but $f(D)$ is not closed

(d) D is closed but $f(D)$ is not bounded

7. Decide if the following functions are uniformily continuous. Prove your answer.

   (a) $f\ (0,1) \to R\ ,\ f(x) = x^2$

   (b) $f\ (0,\infty) \to R\ ,\ f(x) = x^3$

   (c) $f\ (0,\infty) \to R\ ,\ f(x) = 2x + 1$

8. Prove that if $f\ R \to R$, is such that there is a constant $M \neq 0$ such that for all x and y $|f(x) - f(y)| \leq M|x - y|$ then f is uniformily continuous