

## Hw 5

Read chapter 9.1 of the textbook.

Main skills:

- You need to know what absolute convergence is
- You need to know the ratio test
- You need to know the alternating series test

Do the following problems:

1. Decide if the following series are convergent or divergent, explain your reasoning:  
a)  $\sum_{i=1}^{\infty} \ln(i)$    b)  $\sum_{i=1}^{\infty} \frac{i}{3^i}$
2. Decide if the following series are absolutely convergent, convergent but not absolutely convergent or divergent, explain your reasoning:  
a)  $\sum_{i=1}^{\infty} (-1)^i \frac{1}{7i-5}$    b)  $\sum_{i=1}^{\infty} \frac{\sin(i)}{i^2+2}$    c)  $\sum_{i=1}^{\infty} (-1)^i \frac{i^2}{i(i+1)}$    d)  $\sum_{i=1}^{\infty} \frac{(-1)^i}{\sqrt{i+1}}$
3. Prove that if  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  are both absolutely convergent then  $\sum_{n=1}^{\infty} (a_n + b_n)$  is absolutely convergent.
4. For which values of  $x$  does the series  $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$  converge absolutely? For which values of  $x$  does it converge ?.
5. Suppose  $\{a_n\}$  is a decreasing sequence converging to 0 and let  $\{b_n\}$  be the sequence :

$$a_1, a_2, a_3, -a_4, -a_5, -a_6, a_7, a_8, a_9, -a_{10}, -a_{11}, -a_{12} \dots$$

Prove that  $\sum_{n=1}^{\infty} b_n$  converges.