Hw 2

Read chapter 2 of the textbook.
Main skills:

- You need to review induction
- You need to know some useful inequalities and identities.
- You need to know the definition of limit of a sequence, and be able to use it in proofs.
- You need to know the limit laws.

Do the following problems:

1. Prove Bernoulli inequality: $(1+x)^n \geq 1+nx$ for all $n \in N$ and for all $x \in R$, $x > -1$.
2. Consider the following sequences:
   (a) $\{a_n\}$ where :
   $a_1 = \sqrt{2}$
   $a_{n+1} = \sqrt{2+a_n}$.
   Calculate a_3
   (b) $\{b_n\}$ where :
   $b_n = \sum_{i=1}^{n}(i+2)$. Calculate b_4.
   (c) $\{c_n\}$ where :
   $c_n = \sum_{i=1}^{n}(n+2)$. Calculate c_4.
3. Prove that if the sequence $\{a_n\}$ converges to a and c is a constant, then the sequence $\{ca_n\}$ converges to ca.
4. Find the limit c of the following sequences and give a proof the sequence converges to c:
   (a) $\{a_n\}$, where $a_n = \frac{n+1}{n+2}$
   (b) $\{b_n\}$, where $b_n = \frac{1+(-1)^n}{n}$
   (c) $\{c_n\}$, where $c_n = \frac{\sin n}{n}$
   (d) $\{d_n\}$, where $d_n = \frac{2}{n} + (\frac{1}{2})^n$
   (e) $\{e_n\}$, where $e_n = 2\frac{\sin n}{n^2}\frac{n+1}{n+2} + 3$
5. Prove that if $a_n \leq b_n$ for all n, and $\{a_n\}$ converges to a and $\{b_n\}$ converges to b then $a \leq b$