Hw 1

Read chapter 1 of the textbook.

Main skills:

- You need to know the axioms defining R.
- You need to be understand the definition of sup S and inf S and be able to calculate sup S and inf S.
- You need to be able to prove that given values are the sup or inf of a set S

. Do the following problems.

1. For each of the following subsets S of R answer the following questions (no proof needed):

   (a) Is S bounded above ?

   (b) Is S bounded below ?

   (c) If S is bounded above calculate sup S. Is this sup a max ?

   (d) If S is bounded below calculate inf S. Is this inf a min ?

   i) $S = \{1, 2, 3\}$

   ii) $S = \{1 + n \,|\, n \in N\}$

   iii) $S = \{1 + n \,|\, n \in Z\}$

   iv) $S = \{1 + \frac{1}{n} \,|\, n \in N\}$

   v) $S = \{\frac{n+1}{n+2} \,|\, n \in N\}$

2. Prove the following properties of the greatest lower bound of a set: (assume all sets have an inf)

   (a) inf $(A \cup B) =$ min (inf A , inf B)

   (b) inf $(A \cap B) \geq$ max (inf A , inf B)

   (c) if $S \subseteq T$ then inf $T \leq$ inf S.

3. Give an example of sets A and B such that inf $(A \cap B) >$ max (inf A , inf B)

4. Given $S = \{\frac{n}{n+1} \,|\, n \in N\}$ prove that sup S= 1 and inf $S = \frac{1}{2}$

5. Given $S = \{\frac{1}{n} + (-1)^n, \,|\, n \in N\}$ find $i =$ inf S and prove it is the inf, and find $s =$ sup S and prove it is the sup.