Vector spaces : R^n

Subspaces:

Span($v_1, .., v_m$)

Solutions to a homogeneous system —– Kernel of a linear transformation —–- Nullspace of a matrix

Col(A) —–- Range (T) —–- $\{\vec{b}|$ Ax=b has solutions $\}$

row(A)

A set $\{v_1, v_2, ...v_n\}$ of vectors that is linearly independent and spans a vector space V, is called a BASIS for V.

Basis for $R^2 = \{(1,0),(0,1)\}$

Basis for $R^3 = \{(1,0,0),(0,1,0),(0,0,1)\}$

Important facts about bases.

All bases for a given vector space have the same number of elements. This number is called the dimension of the vector space.

If dim(W)=p then

any set of more than p vectors in W is dependent

any set of less than p vectors cannot span W

any set of p linearly independent vectors spans W

any set of p vectors that spans W is independent

any vector in V can be written in a unique way as a linear combination of a basis of V.

$$\vec{v_1} = (1,1,1), \vec{v_2} = (2,3,1), \vec{v_3} = (3,5,1)$$

Find a basis for $Sp\{v_1, v_2, v_3\}$

Method 1:

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$\{v_1, v_2\}$ is our basis.

Method2:

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$\{(1,1,1)(0,1,-1)\}$ is our basis.

How to find a basis for Span $(v_1, v_2, ..., v_n)$.
Method 1:

Write the vectors $v_1, v_2, ..., v_n$ as columns of a matrix A

Reduce A to echelon form AE

Look at the columns of AE that contain a leading term. Pick the corresponding columns of A.

Note : method 1 and 2 in general will produce different basis.

To find the dimension of a space, find a basis and count the elements.

What is the dimension of

Span((1,1,1),(2,3,1),(3,5,1)) ?

Answer: 2.

How to find a basis for Span $(v_1, v_2, ..., v_n)$. Method 2:

Write the vectors $v_1, v_2, ..., v_n$ as rows of a matrix A

Reduce A to echelon form AE

Pick the non zero rows of AE

Assume $v_1, v_2, ..., v_n$ are linearly independent vectors is a space V.

If dim V= n $v_1, v_2, ..., v_n$ are a basis of V

If dim V=$m > n$ we can find m-n vectors to add to $v_1, v_2, ..., v_n$ to form a basis of V

Find a basis for R^3 containing (1,0,-3) and (2,3,8)

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 0 & 1 & 0 \\ -3 & 8 & 0 & 0 & 1 \end{pmatrix}$$

$$AE = \begin{pmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 0 & 1 & 0 \\ 0 & 0 & 3 & -\frac{14}{3} & 1 \end{pmatrix}$$

Basis ((1,0,-3) (2,3,8) (1,0,0))

Alternatively : I know I only need to add one vector, in this case.

Look at

$$A = \begin{pmatrix} 1 & 2 & a \\ 0 & 3 & b \\ -3 & 8 & c \end{pmatrix}$$

$$AE = \begin{pmatrix} 1 & 2 & a \\ 0 & 3 & b \\ 0 & 0 & c + 3a - \frac{14}{3}b \end{pmatrix}$$

choose any vector (a,b,c) with $c + 3a - \frac{14}{3}b \neq 0$

Given a matrix A nullity of A = dimension of N(A).

Find the nullity of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix}$$

A reduces to

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

All solutions of AX=0 are t(1 , -2,1).

so nullity=1. Basis =(1,-2,1)

Find the nullity of A and a basis for N(A) containing (1,1,2,-1,0,0,0) where

$$A = \begin{pmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{pmatrix}$$

$$AE = \begin{pmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

sol in vector form

m(-3,1,0,0,0,0,0) $+$l(-4,0,-2,1,0,0,0)$+$

t(-2,0,0,0,1,0,0)$+$s(0,0,0,0,0,$-\frac{1}{3}$,1)

so nullity is 4

To find basis consider

$$A = \begin{pmatrix} 1 & -3 & -4 & -2 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & -2 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

it reduces to

$$\begin{pmatrix} 1 & -3 & -4 & -2 & 0 \\ 0 & 4 & 4 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{3} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

pivot columns are 1,2,4,5 so the basis we want consistes of the first,second fourth and fifth rows of original matrix A