A **vector space** is a set V together with two operations, called vector addition and scalar multiplication with the following properties: (u, v, w are arbitrary vectors in V, and a, b are scalars.)

1. Associativity of addition

$$u + (v + w) = (u + v) + w$$

.

2. Commutativity of addition

$$v + w = w + v$$

.

3. Identity element of addition There exists an element $0 \in V$, called the zero vector, such that $v + 0 = v$ for all $v \in V$.

4. Inverse elements of addition For all $v \in V$, there exists an element $w \in V$, called the additive inverse of v, such that $v + w = 0$. The additive inverse is denoted $-v$.

5. Distributivity of scalar multiplication with respect to vector addition

$$a(v + w) = av + aw$$

.

6. Distributivity of scalar multiplication with respect to scalar addition

$$(a + b)v = av + bv$$

.

7. Compatibility of scalar multiplication with scalar multiplication $a(bv) = (ab)v$

8. Identity element of scalar multiplication

$$1v = v$$

.

Are the following sets vector spaces?

R^n

All matrices

All mxn matrices

All polynomials

All polynomials of degree 3

Euclidean space R^n

elements of R^n are vectors $(x_1, x_2, ..., x_n)$

For example (1,-1,0) is a vector in R^3

Addition,scalar multiplication.

If $v_1, v_2, \cdots v_k$ are vectors and $c_1, c_2, \cdots c_k$ are scalars then

$$c_1\, v_1 + c_2\, v_2 + \cdot c_k\, v_k$$

is called a LINEAR COMBINATION of $v_1, v_2, \cdots v_k$.

Solutions of a system in vector form

From previous lecture:

$$x_1 = 0.82x_3$$

$$x_2 = 0.92x_3;$$

or

$$(0.82x_3, 0.92x_3, x_3)$$

And the general solution in vector form is

$$x_3(0.82, 0.92, 1)$$

From previous lecture:

$$\begin{aligned} x_1 &= -3x_2 - 4x_4 - 2x_5 \\ x_3 &= -2x_4 \\ x_6 &= \tfrac{1}{3} \end{aligned}$$

or

$$(-3x_2 - 4x_4 - 2x_5, x_2, -2x_4, x_4, x_5, \frac{1}{3})$$

and the general solution in vector form is

$$x_2(-3, 1, 0, 0, 0, 0) + x_4(-4, 0, -2, 1, 0, 0) +$$

$$+ x_5(-2, 0, 0, 0, 1, 0) + (0, 0, 0, 0, 0, \frac{1}{3})$$

Connection between vector equations and systems: Note that the vector equation:

$$x_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + x_3 \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

corresponds to the system:

$$\begin{array}{llll} x_1 + & 2x_2 + & 3x_3 & = b_1 \\ x_1 + & 3x_2 + & 5x_3 & = b_2 \\ x_1 + & x_2 + & x_3 & = b_3 \end{array}$$