

## Math 300 Review

This is not a comprehensive review, but I hope it helps you study for the final. The final will cover chapters 1-9, 14, 15-18, 19 -24 (Fermat 's little th. only) of the textbook

- **The language of math. Chapters 1-4 and 7.** You should be familiar with the idea that we can break up a complicated sentence into "basic statements" linked by connectives: *and, or, not, implies* and quantifiers, *for every, exists*. Typical problems: translate an English sentence into a symbolic statement or vice versa. Given a sentence, write down its negation. Given two statements involving connectives but not quantifiers, decide if they are equivalent. You should know how to use truth tables.
- **Proof techniques. Chapters 1-4** You should know how to do a direct, proof, a proof by contraposition and a proof by contradiction.
- **Induction- Chapter 5 .** You need to be able to do a proof by induction, strong induction and modified induction.
- **Sets Chapters 6-7 .** You should be familiar with the concepts of set, subset, intersection, union and difference of two sets, complement of a set, cartesian product, power set.  
Typical problems: Given two sets A and B, calculate their union, intersection, difference; compute the complement and the power set of a set. Given two sets A and B prove that they are equal, or that one set is a subset of the other.
- **Functions- Chapters 8-9. .** You need to be familiar with the concepts of injective, surjective, bijective function; inverse of a function, composition of two functions. Typical problem: decide if a given function is injective, surjective, bijective. Given a function, compute its inverse; given two functions compute their composition.
- **Cardinality. Chapter 14.** You should be familiar with the concept of denumerable set and with Cantor diagonalization proof that  $\mathbb{R}$  is not denumerable. Typical problem: given a set A prove it is denumerable, or that it is not denumerable.
- **Arithmetic : chapters 15-18, 23** You should be familiar with the division theorem (we did not prove it ), the definition of greatest common divisor, the euclidean algorithm, diophantine equations, unique prime factorization theorem. Typical computational problems: Given  $a$  and  $b$  in  $\mathbb{Z}$  find  $\gcd(a, b)$ . Solve a linear diophantine equation. Other problems: prove properties of gcd. Divisibility problems.
- **Modular arithmetic. Chapters 19-21 and 24** You need to be familiar with the notions of congruence mod  $m$ , and congruence class,  $\mathbb{Z}_m$ ,

inverse of an element in  $Z_m$ . Fermat's little theorem. Typical computational problems: solve a linear congruence, compute mod  $m$ . Other problems may ask you to prove properties of congruences or of  $Z_m$ , or to use congruences in proofs, for example in divisibility proofs.

- **Equivalence relations and equivalence classes. Chapter 22** You need to be able to decide whether a relation  $R$  is an equivalence relation or not, and if it is, you need to be able to describe its equivalence classes.