SPRING 2019 MATH 300 A FINAL EXAM

Write clearly and legibly. Justify all your answers.

You will be graded for correctness and clarity of your solutions.

You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides. You may use a calculator.

You can use elementary algebra and any result that we proved in class (but not in the homework). You need to prove everything else.

Please raise your hand and ask a question if anything is not clear.

This exam contains 8 pages, please make sure you have a complete exam. You have 1 hr and 50 minutes. Good luck

NAME:_________

PROBLEM 1 (10 points) _______

PROBLEM 2 (8 points) _______

PROBLEM 3 (10 points) ________

PROBLEM 4 (16 points) ________

PROBLEM 5 (8 points) ________

PROBLEM 6 (8 points) ________

PROBLEM 7 (10 points) ________

Total (70 points) ____________

- **Problem 1** Given sets A, B, prove that $(A - (A - B)) \subseteq B$. Give an example to show that equality does not have to hold.

- **Problem 2** Write a statement equivalent to the negation of

$$\exists x \in A \, \forall y \in B \, (x \leq y) \Rightarrow (\exists z \in C \, ((z > x) \Rightarrow (z > y \wedge z = y)))$$

that does not use the negation symbol $\neg$. You are allowed to use $\neq$.

- **Problem 3** Prove that the sum of two odd perfect squares is never a perfect square. A perfect square is an integer z such that $z = k^2$ for some integer k.

- **Problem 4** Define a function $f : Z \to Z$ by:

$$f(x) = \begin{cases} x+2 & \text{if 3 divides } x \\ x-1 & \text{otherwise} \end{cases}$$

1. Is f injective ? *(Give a proof).*

2. Is f surjective ? *(Give a proof).*

3. Prove that $\forall n \in Z^+ \forall m \in Z f^{3n}(m) = m$ (Recall that f^n means f composed with itself n times, so for example $f^2(x) = f(f(x))$)

- **Problem 5** Find all integer solutions of $3 \cdot 7^{1022} x \equiv 25 \bmod 31$

- **Problem 6** Show that the relation r defined on R, the set of real numbers by xry iff $x - y \in Z$ is an equivalence relation.

- **Problem 7** Prove that if A and B are denumerable sets, then their cartesian product $A \times B$ is denumerable. For this problem you can assume that Z, EVEN (the set of even integers), ODD (the set of odd integers) , $Z^+ \times Z^+$, $Z \times Z$ are denumerable , without having to prove it.