Spring 2019 Math 300 Midterm exam

Write clearly and legibly. Justify all your answers.

You will be graded for correctness and clarity of your solutions.

You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides. You may use a calculator.

You can use elementary algebra and any result that we proved in class (but not in the homework). You need to prove everything else.

Please raise your hand and ask a question if anything is not clear.

This exam contains 6 pages please make sure you have a complete exam. You have 50 minutes. Good luck

NAME:_____

NAME:_____

PROBLEM 1 _____

PROBLEM 2 _____

PROBLEM 3 _____

PROBLEM 4 _____

Total _____

- **Problem 1** Recall that the Fibonacci sequence is defined by:

  $u_1 = 1$

  $u_2 = 1$

  $u_{n+1} = u_n + u_{n-1}$ if $n + 1 \geq 3$

  Prove that $\sum_{i=1}^{n} u_i = u_{n+2} - 1$

- **Problem 2** Let A_i be the set of all positive integers divisible by i, that is $A_i = \{n \in Z^+ | i \text{ divides } n\}$.

  1. Prove that $A_2 \cap A_3 = A_6$

  2. Prove that for all $k \geq 2$ the set $\bigcap_{i=2}^{k} A_i$ is infinite.

  3. Is $\bigcap_{i=2}^{\infty} A_i$ infinite ? Justify your answer.

- **Problem 3**

  Define a function $f : \mathrm{Z} \to \mathrm{Z}$ by:

$$f(x) = \begin{cases} x + 2 & \text{if } x \geq 0 \\ x + 5 & \text{if } x < 0 \end{cases}$$

  1. Is f injective ? *(Give a proof).*

  2. Is f surjective ? *(Give a proof).*

- **Problem 4** For each statement below circle if it is true or false and give a proof. $EVEN^+$ is the set of even positive integers.

  1. $\forall y \in EVEN^+ \, \exists x \in Z \quad y = \frac{x^2}{2}$ TRUE FALSE

  2. $\forall x \in EVEN^+ \, \exists y \in Z \quad y = \frac{x^2}{2}$ TRUE FALSE

  3. $\exists x \in Z \, \forall y \in Z \quad y + x > 3$ TRUE FALSE