Spring 2018 Math 300 Midterm exam

Write clearly and legibly. Justify all your answers.

You will be graded for correctness and clarity of your solutions.

You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides. You may use a calculator.

You can use elementary algebra and any result that we proved in class (but not in the homework). You need to prove everything else.

Please raise your hand and ask a question if anything is not clear.

This exam contains 7 pages and is worth a total of 45 points.

You have 50 minutes. Good luck

NAME:_____

NAME:_____

PROBLEM 1 (12 points) _____

PROBLEM 2 (9 points) _____

PROBLEM 3 (8 points) _____

PROBLEM 4 (8 points) _____

PROBLEM 5 (8 points) _____

Total _____

- **Problem 1** In this problem 3 div a means *3 divides a* and 3 notdiv a means *3 does not divide a.*
  - Write a statement equivalent to the negation of

    $$\forall x \in Z, \forall y \in Z,\ 3 \text{ div } xy \Leftrightarrow (3 \text{ div } x \vee 3 \text{ div } y)$$

    that does not contain the negation symbol $\neg$ (not). You are allowed to use notdiv.

  - Prove or disprove that

    $$\forall x \in Z, \forall y \in Z,\ 3 \text{ div } xy \Leftrightarrow (3 \text{ div } x \vee 3 \text{ div } y)$$

  - Prove or disprove that

    $$\forall x \in Z, \forall y \in Z,\ 3 \text{ div } xy \Leftrightarrow (3 \text{ div } x \wedge 3 \text{ div } y)$$

- **Problem 2** Define a function $f : Z \to Z$ by:

$$f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ x-1 & \text{if } x \text{ is odd and } x \geq 0 \\ x-3 & \text{if } x \text{ is even and } x \geq 0 \end{cases}$$

a) Is f injective ? *(Give a proof).*

b) Is f surjective ? *(Give a proof).*

- **Problem 3** Use induction to prove that $\sum_{i=1}^{2n}(-1)^i i = n$

- **Problem 4** For each of the following statements circle whether the statement is true or false and give a proof.

  1. $\exists x \in Z, \quad \forall y \in Z,\ x - y = 10$.

     TRUE FALSE

  2. $\forall x \in Z, \quad \exists S \in P(Z), \forall w \in S,\ x + w = 0$ (Here $P(Z)$ is the power set of Z) .

     TRUE FALSE

- **Problem 5** Let A, B , C be sets. Prove that

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$