Spring 2018 Math 300 A Final exam

Write clearly and legibly. Justify all your answers.

You will be graded for correctness and clarity of your solutions.

You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides. You may use a calculator.

You can use elementary algebra and any result that we proved in class (but not in the homework). You need to prove everything else.

Please raise your hand and ask a question if anything is not clear.

This exam contains 9 pages and is worth a total of 80 points. You have 1 hr and 50 minutes. Good luck

NAME:_________

PROBLEM 1 (10 points) _______

PROBLEM 2 (8 points) _______

PROBLEM 3 (14 points) ________

PROBLEM 4 (10 points) ________

PROBLEM 5 (12 points) ________

PROBLEM 6 (8 points) ________

PROBLEM 7 (10 points) ________

PROBLEM 8 (8 points) ________

Total ____________

- **Problem 1** Given sets A, B, C in some universe U prove that

$$((A \cup B) - C)^c = (A - C)^c \cap (B - C)^c.$$

- **Problem 2** Write a statement equivalent to the negation of

$$\forall x \in A\, \exists y \in B\, (x > y) \Rightarrow (\exists z \in C\, ((z > x) \Rightarrow (z > y \vee z = 0)))$$

that does not use the negation symbol $\neg$. You are allowed to use $\neq$

- **Problem 3** Prove that $\forall x \in Z$, 3 div $x \Leftrightarrow 3$ div x^3

Use the result above to prove that $\sqrt[3]{3}$ is irrational

- **Problem 4** Prove that the set A $=\{S | S \subseteq N$ and $|S| \leq 1\}$ (that is A is the set of all subsets of N that have zero or one element) is denumerable.

- **Problem 5** Prove that if p and q are distinct primes then
  $\forall a \in Z, \forall b \in Z,\ a \equiv b \bmod pq \Leftrightarrow (a \equiv b \bmod p \wedge a \equiv b \bmod q)$.

  Give a counterexample to show the theorem is not true when p or q are not primes.

- **Problem 6** Find all integer solutions of $3^{122}x \equiv 5 \bmod 11$

- **Problem 7** Consider the relation R on Z defined by

$$aRb \text{ iff } 3a + 3b \equiv 0 \bmod 6$$

Prove R is an equivalence relation.

List and describe all equivalence classes for R. Remember to justify your answer.

- **Problem 8** Consider the sequence $\{a_n\}$ defined by:

  $a_1 = 2$

  $a_2 = 3$

  $a_3 = 2$

  $a_{n+1} = 4a_n - 5a_{n-1} + 2a_{n-2}$ if $n + 1 \geq 4$

  Prove that $\forall n \geq 1$, $a_n = 3n + 1 - 2^n$