

Math 300 Spring 2017 Midterm Exam

*Write clearly and legibly. Justify all your answers.*

*You will be graded for correctness and clarity of your solutions.*

*You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides.*

*You may use a calculator.*

*You can use elementary algebra and any result that we proved in class. You need to prove everything else.*

*Please raise your hand and ask a question if anything is not clear.*

*This exam contains 5 pages and is worth a total of 50 points.*

*You have 50 minutes. Good luck*

NAME:-----

PROBLEM 1 (10) -----

PROBLEM 2(10) -----

PROBLEM 3 (10) -----

PROBLEM 4 (10)-----

PROBLEM 5 (10) -----

Total -----

**Problem 1:** Let  $A$  and  $B, C$  be sets.

1. (5 points) Prove that  $(A - B) \cap (A - C) \subseteq A - (B \cap C)$

2. (7 points) Is  $\forall A, B, C \quad (A - B) \cap (A - C) = A - (B \cap C)$  true? Justify your answer.

**Problem 2** (10 points) Prove that  $\forall x \in \mathbb{Z} \ 14 \mid x \Leftrightarrow (2 \mid x \wedge 7 \mid x)$

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let is find  $f(n)$   
s.t.  $1+3+\dots+(2n+1) =$   
 $\delta(n)$

**Problem 3**(10 points) Guess a formula for  $1 + 3 + 5 + \dots + (2n + 1)$ , the sum of the first  $n$  odd positive integers and use induction to prove your formula is correct.

**Problem 4** Define a function  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  by:

$$f(x) = \begin{cases} x - 3 & \text{if } x \geq 0 \\ x + 5 & \text{if } x < 0 \end{cases}$$

1. (5 points) Is  $f$  injective ? Prove your answer.

2. (5 points) Is  $f$  surjective ? Prove your answer.

**Problem 5**(10 points) Let  $A$  be the set of all functions from  $Z$  to  $Z$ . For each statement below , write the negation of the statement and prove whether the original statement(NOT the negation) is true or false.

(a)  $\forall f \in A \exists g \in A \forall x \in Z g(x) \geq f(x)$ .

NEGATION:

True or false ? Give a proof.

(b)  $\exists f \in A \forall g \in A \forall x \in Z g(x) \geq f(x)$ .

NEGATION:

True or false ? Give a proof.

(c)  $\exists f \in A \forall y \in Z \exists x \overset{z}{\underbrace{y \text{ odd}}}(x) \Rightarrow (x \text{ even} \wedge f(x) = y)$ .

NEGATION:

True or false ? Give a proof.