Math 300 Spring 2017 Final Exam

You can use any result we proved in class and elementary algebra. You need to prove everything else.
Write clearly and legibly. Prove all your answers.
You will be graded for correctness and clarity of your solutions.
You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides. You may use a calculator.
This exam contains 8 pages and is worth a total of 80 points.
You have one hour and 50 minutes. Good luck

NAME:_________ STUDENT ID NUMBER: _________

PROBLEM 1 _______

PROBLEM 2 _______

PROBLEM 3 ________

PROBLEM 4 ________

PROBLEM 5 ________

PROBLEM 6 ________

PROBLEM 7 ________

Total ____________

Problem 1 (10 points)Prove that , $\forall x \in Z$, 7 div $x \Leftrightarrow$ 7 div x^2.

Problem 2(10 points) Compute $(20 * (16)^{40} + 13^{1600}) \bmod 17$.

Find the unit digit of 7^{1000}.

Problem 3 (10 points) Let EVEN= $\{x \in Z^+ \mid x \text{ is even }\}$ and ODD=$\{x \in Z^+ \mid x \text{ is odd }\}$. Prove EVEN $\times$ ODD is denumerable. You may assume Z, EVEN, ODD, $Z \times Z$, $Z^+ \times Z^+$ are denumerable.

Problem 4 (10 points) Prove that a every prime number greater than 3 is of the form 6n+1 or 6n -1 for some n in Z^{+}.

Problem 5 (10 points) List all the elements of $P(\{1\} \times \{a, b\})$.

Problem 6 (10 points) Find all integers x that satisfy $36x \equiv 30 \bmod 42$.

Find the inverse of 5 in Z_7.

Problem 7(10 points) Remember from calculus that the $(\sin(x))^{'} = \cos(x)$ and $(\cos(x))^{'} = -\sin(x)$, that is the derivative of $\sin(x)$ is $\cos(x)$ and the derivative of $\cos(x)$ is $-\sin(x)$. Use induction to prove that , for any $n \in Z^{+}$,the nth derivative of $\sin(x)$ is given by the formula :

$$(\sin(x))^{(n)} = \begin{cases} \cos(x) & \text{if } n \equiv 1 \bmod 4 \\ -\sin(x) & \text{if } n \equiv 3 \bmod 4 \\ -\cos(x) & \text{if } n \equiv 2 \bmod 4 \\ \sin(x) & \text{if } n \equiv 0 \bmod 4 \end{cases}$$

Problem 8 Let R be the relation on Z defined by aRb iff $a - b$ is even. Prove R is an equivalence relation and describe its equivalence classes.