Math 300 Fall 2016 Final Exam

You can use any result we proved in class and elementary algebra. You need to prove everything else.
Write clearly and legibly. Prove all your answers.
You will be graded for correctness and clarity of your solutions.
You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides. You may use a calculator.
This exam contains 9 pages and is worth a total of 90 points.
You have one hour and 50 minutes. Good luck

FIRST AND LAST NAME:_________

PROBLEM 1 _______

PROBLEM 2 _______

PROBLEM 3 ________

PROBLEM 4 ________

PROBLEM 5 ________

PROBLEM 6 ________

PROBLEM 7 ________

PROBLEM 8 ________

Total ____________

Problem 1 (10 points) Assume P, Q and R are statements. Is $\neg P \vee \neg Q \vee R$ equivalent to $P \Rightarrow (Q \Rightarrow R)$? Justify your answer.

Problem 2(10 points) Given the following recursive definition:

$a_1 = 1$

$a_{n+1} = 3a_n - 1$ for $n \geq 1$

Prove that $a_n = \frac{3^{n-1}+1}{2}$

Problem 3 (10 points) Assume A is a denumerable set .
Let $B = \{x \in Z^+ \mid x \geq 10\}$ Prove $A \cup B$ is denumerable. You may assume A and B are disjoint. You may also assume Z, $Even_{Z^+}$, $Even_Z$, Odd_{Z^+}, Odd_Z, $Z \times Z$, $Z^+ \times Z^+$ are denumerable, but you cannot assume the result from the hw that the union of two denumerable sets is denumerable without reproving it.

Problem 4 (10 points) Prove that no integer of the form $4n+3$ is the sum of two squares (that is the sum of the squares of two integers).

Problem 5 (10 points) Prove that for any sets A,B and C
$A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Problem 6

(5 points) Find all integers x that satisfy $12x \equiv 24 \bmod 30$.

(5 points) Compute $2^{55} \bmod 53$. (Note : 53 is prime)

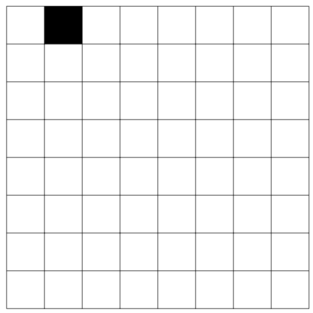
(5 points) List all invertible elements of Z_{18}.

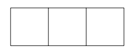
Problem 7(15 points) Given $f : A \to A$ and $g : A \to A$.

1. Prove that if f and g are both injective, then $g \circ f$ is injective

2. Is it true that if $g \circ f$ is injective then f and g are both injective? Prove your answer.

Problem 8 Prove that an 8x8 checkerboard with the square in position 1, 2 i.e top row , second from the left removed cannot be covered by 1x3 tiles. A covering must be such that each square of a 1x3 tiles covers exactly one square of the board.





1x3 tile

checkerboard with square (1,2) removed