Math 300 Fall 2016 Midterm Exam

Write clearly and legibly. Justify all your answers.
You will be graded for correctness and clarity of your solutions.
You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides. You may use a calculator.
You can use elementary algebra and any result that is proved in chapters 1-9 of the textbook (but not in the exercises). You need to prove everything else.
Please raise your hand and ask a question if anything is not clear.
This exam contains 5 pages and is worth a total of 40 points. You have 50 minutes. Good luck

NAME:__________

PROBLEM 1 _______

PROBLEM 2 _______

PROBLEM 3 ________

PROBLEM 4 ________

Total ____________

- **Problem 1** (10 points) Prove that $\sum_{i=1}^{2n} \frac{(-1)^{i+1}}{i} = \sum_{i=n+1}^{2n} \frac{1}{i}$, for all $n \in Z^+$.

By induction

Base case: if $n=1$ $\sum_{i=1}^{2} \frac{(-1)^{i+1}}{i} = 1 - \frac{1}{2} = \frac{1}{2}$ and $\sum_{i=2}^{2} \frac{1}{i} = \frac{1}{2}$

so identity is true for $n=1$

Induction step: assume $\sum_{i=1}^{2k} \frac{(-1)^{i+1}}{i} = \sum_{i=k+1}^{2k} \frac{1}{i}$ then

$$\sum_{i=1}^{2(k+1)} \frac{(-1)^{i+1}}{i} = \sum_{i=1}^{2k} \frac{(-1)^{i+1}}{i} + \frac{1}{2k+1} - \frac{1}{2k+2} = \sum_{i=k+1}^{2k} \frac{1}{i} + \frac{1}{2k+1} - \frac{1}{2k+2} =$$

$$= \sum_{i=(k+1)+1}^{2k} \frac{1}{i} + \frac{1}{k+1} + \frac{1}{2k+1} - \frac{1}{2k+2} = \sum_{i=(k+1)+1}^{2k+1} \frac{1}{i} + \frac{2}{2(k+1)} - \frac{1}{2k+2} =$$

$$= \sum_{i=(k+1)+1}^{2k+1} \frac{1}{i} + \frac{1}{2k+2} = \sum_{i=(k+1)+1}^{2(k+1)} \frac{1}{i}$$

so if the identity holds for k, it holds for $k+1$

- **Problem 2** Define a function $f : Z^+ \to Z^+$ by:

$$f(n) = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ 2n-1 & \text{if } n \text{ is even} \end{cases}$$

a)(5 points) Is f injective ? *(Give a proof).*

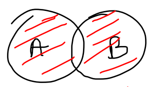
yes if $n_1, n_2 \in Z^+$ and $n_1 \neq n_2$ and
n_1 and n_2 both odd then certainly $n_1+1 \neq n_2+1$ so $f(n_1) \neq f(n_2)$
n_1 and n_2 both even then certainly $2n_1-1 \neq 2n_2-1$ so $f(n_1) \neq f(n_2)$
n_1 odd and n_2 even (or viceverse) then $f(n_1)$ is even and $f(n_2)$ is odd (or viceverse) so $f(n_1) \neq f(n_2)$

b) (5 points) Is f surjective ? *(Give a proof).*

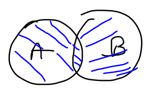
No look at $y=1$ then 1 is odd so if $1 = f(n)$
then n is even and $1 = 2n-1$, but the only solution
is $n=1$ and $f(1)=2$
so there is no $n \in Z^+$ s.t $f(n)=1$

- **Problem 3** Let A and B be sets.
  1. (5 points) Prove that $(A - B) \cup (B - A) \subseteq A \cup B$
  2. (5 points) Is it true or false that $(A - B) \cup (B - A) = A \cup B$? Prove your answer.

1) Assume $x \in (A-B) \cup (B-A)$, then $x \in A-B$ so $x \in A$, therefore $x \in A \cup B$, or $x \in B-A$ so $x \in B$ and therefore $x \in A \cup B$
In any case if $x \in (A-B) \cup (B-A)$ then $x \in A \cup B$

2) 

$(A-B) \cup (B-A)$



$A \cup B$

No if for exemple $A = \{1, 2\}$ $B = \{2, 3\}$
$A - B = \{1\}$, $B - A = \{3\}$ $(A-B) \cup (B-A) = \{1, 3\}$ but
$A \cup B = \{1, 2, 3\}$

- **Problem 4**(10 points) For each of the following statements circle whether the statement is true or false and give a proof.

  1. $\forall x \in Z, \quad \exists y \in P(Z), \quad x \in y$.

     TRUE FALSE

     Given $x \in Z$ take $y = \{x\}$ then $y \in P(Z)$ and $x \in y$

  2. $\forall x \in Z \quad \exists y \in Z \quad \forall w \in Z \quad xy\cancel{w} \geq \cancel{x}w$

     TRUE FALSE

We shall prove $\exists x \in Z \ \forall y \in Z \ \exists w \in Z \quad xy < w$

Take $x = 1$ then given y take $w = y+1$ then $y < y+1$ is true