Math 300 Fall 2016 Midterm Exam

Write clearly and legibly. Justify all your answers.

You will be graded for correctness and clarity of your solutions.

You may use one 8.5 x 11 sheet of notes; writing is allowed on both sides. You may use a calculator.

You can use elementary algebra and any result that is proved in chapters 1-9 of the textbook (but not in the exercises). You need to prove everything else.

Please raise your hand and ask a question if anything is not clear.

This exam contains 5 pages and is worth a total of 40 points. You have 50 minutes. Good luck

NAME:_____

PROBLEM 1 _____

PROBLEM 2 _____

PROBLEM 3 _____

PROBLEM 4 _____

Total _____

- **Problem 1** (10 points) Prove that $\sum_{i=1}^{2n} \frac{(-1)^{i+1}}{i} = \sum_{i=n+1}^{2n} \frac{1}{i}$, for all $n \in Z^+$.

- **Problem 2** Define a function $f : Z^+ \rightarrow Z^+$ by:

$$f(n) = \begin{cases} n + 1 & \text{if } n \text{ is odd} \\ 2n - 1 & \text{if } n \text{ is even} \end{cases}$$

a)(5 points) Is f injective ? *(Give a proof)*.

b) (5 points) Is f surjective ? *(Give a proof)*.

- **Problem 3** Let A and B be sets.

  1. (5 points) Prove that $(A - B) \cup (B - A) \subseteq A \cup B$
  2. (5 points) Is it true or false that $(A - B) \cup (B - A) = A \cup B$? Prove your answer.

- **Problem 4**(10 points) For each of the following statements circle whether the statement is true or false and give a proof.

  1. $\forall x \in Z, \quad \exists y \in P(Z), \quad x \in y$.

     TRUE   FALSE

  2. $\forall x \in Z \quad \exists y \in Z \quad \forall w \in Z \quad xy \geq w$

     TRUE   FALSE