

## Hw 7

Read chapter 16,17,18,19,20 of the textbook.

You need to know:

- how to compute mod  $m$ .
- how to compute in  $Z_m$
- the Euclidean algorithm
- how to solve a linear diophantine equation in two variables
- how to solve a linear congruence

Do the following problems from your textbook:

- p. 206: 16.1
- p. 215: 17.1
- p. 224: 18.1
- p.249 : 20.2 i)
- p. 271: 1,2,4
- p. 272: 7

. Do the following additional problems.

1. Compute  $(8 \cdot 102 + 24) \bmod 5$ ,  $(-11 + 6^5 + (-1)^{20}) \bmod 5$ ,  $8^{300} \bmod 7$  and  $6^{99} \bmod 7$
2. Write down the addition and multiplication tables for  $Z_3$  and  $Z_4$ .
3. Use problem 2 pag 271 to argue that the diophantine equation  $x^2 - 7y^2 = 67$  has no solutions.
4. Let  $u_n$  be the  $n$ th Fibonacci number. Prove that for  $n \geq 2$  the Euclidean algorithm takes exactly  $n-1$  steps to compute  $(u_{n+1}, u_n)$ . Here a step is just one division with remainder, that is a line in the computation.
5. Is it possible to write 15 as an integer linear combination of 96 and 117? If it is find  $a$  and  $b$  in  $Z$  such that  $96a + 117b = 15$ . if it is not, explain why not.