Hw 5

Read chapter 14 of the textbook.

Main skills:

- You need to be able to prove that a given infinite set is denumerable or not denumerable.

Do the following problems from your textbook:

- p. 181 14.1, 14.2 (you may assume $A \cap B = \emptyset$ for both problems).
- 14.3 (you may want to use additional problem 4).

. Do the following additional problems.

1. Prove that the set $A = \{ x \in Z^+ : 7 \text{ divides } x \}$ is denumerable.
2. Prove that the set $A = \{ x \in Z : 7 \text{ divides } x \}$ is denumerable.
3. Prove that the interval $[1, 3]$ is not denumerable.
4. In a previous homework you have shown that any $n \in Z^+$ can be written in the form $n = 2^m \cdot h$, where $m \geq 0$ and h is odd.
   a) Prove that m and h are unique, that is prove that
   $$n = 2^{m_1} \cdot h_1 = 2^{m_2} \cdot h_2 \Rightarrow (m_1 = m_2) \wedge (h_1 = h_2)$$
   This allows you to define a function $f : Z^+ \to (Z^+ \cup \{0\}) \times ODD_{Z^+}$ with $f(n) = (m, h)$, where $n = 2^m h$.
   b) Prove that f is a bijection.
   c) Prove that $Z^+ \times Z^+$ is denumerable by defining a bijection g : $Z^+ \times Z^+ \to (Z^+ \cup \{0\}) \times ODD_{Z^+}$.
5. Let A be a denumerable set and $a \in A$. Prove that $A - \{a\}$ is denumerable.
6. Prove that P(Z^+) is not denumerable: by contradiction assume $f : Z^+ \to P(Z^+)$ is a bijection.
   As an example, consider the following table

		1	2	3	4	5	$\cdots$
$f(1)$		T	T	F	T	F	$\cdots$
$f(2)$		F	F	F	T	F	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$f(n)$		F	T	T	T	T	$\cdots$

   In each row i we have T in the column corresponding to j iff $j \in f(i)$, we have F in the column corresponding to j iff $j \notin f(i)$, so for example, if you look at the first row $1 \in f(1)$, $2 \in f(1)$, $3 \notin f(1)$, $\cdots$.

   Use a diagonalization argument similar to Cantor's argument in the proof that R is not denumerable, to find a subset S of Z^+ that is different form $f(i)$ for all i.

   Then use S to finish your proof by contradiction.